Transformers And Induction Machines

Syllabus

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PART - A

UNIT - 1

**BASIC CONCEPTS**: Principle of operation of transformer, Constructional details of shell type and core type single-phase and three-phase transformers. EMF equation, operation of practical power transformer under no load and on load (with phasor diagrams). Concept of ideal transformers, current inrush in transformers.  

6 Hours

UNIT- 2


6 Hours

UNIT- 3

Parallel operation -need, conditions to be satisfied for parallel operation. Load sharing in case of similar and dissimilar transformers. Auto-transformers, copper economy. Brief discussion on constant voltage transformer, constant current transformer.  

6 Hours
UNIT- 4


8 Hours

PART - B

UNIT -5


7 Hours

UNIT- 6


6 Hours

UNIT -7


6 Hours
UNIT- 8

(a) Starting and speed Control of Three-phase Induction Motors: Need for starter. Direct on line (DOL), Star-Delta and autotransformer starting. Rotor resistance starting. Soft(electronic) starters. Speed control -voltage, frequency, and rotor resistance. 4 Hours

(b) Single-phase Induction Motor: Double revolving field theory and principle of operation. Types of single-phase induction motors: split-phase, capacitor start, shaded pole motors. Applications. 3 Hours
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PART - A

UNIT - 1

BASIC CONCEPTS: Principle of operation of transformer, Constructional details of shell type and core type single-phase and three-phase transformers. EMF equation, operation of practical power transformer under no load and on load (with phasor diagrams). Concept of ideal transformers, current inrush in transformers.  

Transformers: The static electrical device which transfers the voltage from one level to another level by the principle of self and mutual induction without change in frequency.

Michael Faraday propounded the principle of electro-magnetic induction in 1831. It states that a voltage appears across the terminals of an electric coil when the flux linked with the coil changes. The magnitude of the induced voltage is proportional to the rate of change of the flux linkages. This finding forms the basis for many magneto electric machines.

The earliest use of this phenomenon was in the development of induction coils. These coils were used to generate high voltage pulses to ignite the explosive charges in the mines. As the d.c. power system was in use at that time, very little of transformer principle was made use of. In the d.c. supply system the generating station and the load center have to be necessarily close to each other due to the requirement of economic transmission of power.

Transformers can link two or more electric circuits. In its simple form two electric circuits can be linked by a magnetic circuit, one of the electric coils is used for the creation of a time varying magnetic field. The second coil which is made to link this field has an induced voltage in the same. The magnitude of the induced emf is decided by the number of turns used in each coil. Thus the voltage level can be increased or decreased by changing...
the number of turns. This excitation winding is called a primary and the output winding is called a secondary. As a magnetic medium forms the link between the primary and the secondary windings there is no conductive connection between the two electric circuits. The transformer thus provides an electric isolation between the two circuits. The frequency on the two sides will be the same. As there is no change in the nature of the power, the resulting machine is called a ‘transformer’ and not a ‘converter’. The electric power at one Voltage/current level is only ‘transformed’ into electric power, at the same frequency, to another voltage/current level.

Even though most of the large-power transformers can be found in the power systems, the use of the transformers is not limited to the power systems. The use of the principle of transformers is universal. Transformers can be found operating in the frequency range starting from a few hertz going up to several mega hertz. Power ratings vary from a few miliwatts to several hundreds of megawatts. The use of the transformers is so widespread that it is virtually impossible to think of a large power system without transformers. Demand on electric power generation doubles every decade in a developing country. For every MVA of generation the installed capacity of transformers grows by about 7MVA.

**Classification of Transformer:**

The transformers are classified according to:

1. The Type of Construction:
   (a) Core Type Transformer
   (b) Shell Type Transformer
2. The Number of Phases:
   (a) Single Phase Transformer
   (b) Three Phase Transformer
3. The Placements:
   (a) Indoor Transformer
   (b) Outdoor Transformer
4. The Load:
   (a) Power Transformer
   (b) Distribution Transformer
Ideal Transformer

To understand the working of a transformer it is always instructive, to begin with the concept of an *ideal* transformer with the following properties.

1. Primary and secondary windings have no resistance.

2. All the flux produced by the primary links the secondary winding i.e., there is no leakage flux.

3. Permeability $\mu_r$ of the core is infinitely large. In other words, to establish flux in the core vanishingly small (or zero) current is required.

4. Core loss comprising of *eddy current* and *hysteresis* losses are neglected.

Construction of a Transformer

There are two basic parts of a transformer:

1. Magnetic core
2. Winding or coils

- **MAGNETIC CORE:** The core of a transformer is either square or rectangular in size. It is further divided in two parts. The vertical portion on which the coils are bound is called limb, while the top and bottom horizontal portion is called yoke of the core as shown in fig. 2.

![Fig. 2](image-url)

Core is made up of laminations. Because of laminated type of construction, eddy current losses get minimized. Generally high grade silicon steel laminations (0.3 to 0.5 mm thick) are used. These laminations are insulated from each other by using insulation like varnish. All laminations are varnished. Laminations are overlapped so that to avoid the air
gap at the joints. For this generally ‘L’ shaped or ‘I’ shaped laminations are used which are shown in the fig. 3 below.

![Fig. 3](image)

**WINDING:** There are two windings, which are wound on the two limbs of the core, which are insulated from each other and from the limbs as shown in fig. 4. The windings are made up of copper, so that, they possess a very small resistance. The winding which is connected to the load is called secondary winding and the winding which is connected to the supply is called primary winding. The primary winding has $N_1$ number of turns and the secondary windings have $N_2$ number of turns.

![Fig. 4. Single Phase Transformer](image)
**TYPES OF TRANSFORMERS:**

The classification of transformer is based on the relative arrangement or disposition of the core and the windings. There are two main types of transformers.

1. Core type
2. Shell type

**CORE TYPE:**

Fig 5(a)& (b) shows the simplified representation of a core type transformer, where the primary and secondary winding have been shown wound on the opposite sides. However, in actual practise, half the primary and half the secondary windings are situated side by side on each limb, so as to reduce leakage flux as shown in fig 6. This type of core construction is adopted for small rating transformers.
SHELL TYPE:

In this type, the windings occupy a smaller portion of the core as shown in fig 5. The entire flux passes through the central part of the core, but outside of this a central core, it divides half, going in each direction. The coils are form wound, multilayer disc-type, each of the multilayer discs is insulated from the other by using paper. This type of construction is generally preferred for high voltage transformers.

Fig. 7 (a) & (b) Single Phase Shell Type Transformer
A single phase transformer works on the principle of mutual induction between two magnetically coupled coils. When the primary winding is connected to an alternating voltage of r.m.s value, $V_1$ volts, an alternating current flows through the primary winding and sets up an alternating flux $\phi$ in the material of the core. This alternating flux $\phi$, links not only the primary windings but also the secondary windings. Therefore, an e.m.f $e_1$ is induced in the primary winding and an e.m.f $e_2$ is induced in the secondary winding, $e_1$ and $e_2$ are given:

$$e_1 = -N_1 \frac{d\phi}{dt} \quad \text{(a)}$$

$$e_2 = -N_2 \frac{d\phi}{dt} \quad \text{(b)}$$

If the induced e.m.f is $e_1$ and $e_2$ are represented by their rms values $E_1$ and $E_2$ respectively, then:

$$E_1 = -N_1 \frac{d\phi}{dt} \quad \text{(1)}$$

$$E_2 = -N_2 \frac{d\phi}{dt} \quad \text{(2)}$$
Therefore, \( \frac{E_2}{E_1} = \frac{N_2}{N_1} = k \) \( \cdots \) (3)

K is known as the transformation ratio of the transformer. When a load is connected to the secondary winding, a current \( I_2 \) flows through the load, \( V_2 \) is the terminal voltage across the load. As the power transfered from the primary winding to the secondary winding is same,

Power input to the primary winding = Power output from the secondary winding.

\( E_1I_1 = E_2I_2 \)

(Assuming that the power factor of the primary is equal to the secondary).

Or, \( \frac{E_2}{E_1} = \frac{n_1}{n_2} = k \) \( \cdots \) (4)

From eqn (3) and (4), we have

\[ \frac{E_2}{E_1} = \frac{N_2}{N_1} = \frac{n_1}{n_2} = k \] \( \cdots \) (5)

The directions of emf’s \( E_1 \) and \( E_2 \) induced in the primary and secondary windings are such that, they always oppose the primary applied voltage \( V_1 \).

**EMF Equation of a transformer:**

Consider a transformer having,

\( N_1 \) = Primary turns
\( N_2 \) = Secondary turns
\( \Phi_m = \) Maximum flux in the core
\( \Phi_m = B_m \times A \) webers
\( f = \) frequency of ac input in hertz (Hz)
The flux in the core will vary sinusoidally as shown in figure, so that it increases from zero to maximum “$\phi_m$” in one quarter of the cycle i.e, $\frac{1}{4f}$ second

Therefore, average rate of change of flux = $\frac{\phi_m}{1/4f}$

= $4f\phi_m$

We know that, the rate of change of flux per turn means that the induced emf in volts.

Therefore, average emf induced per turn = $4f\phi_m$ volts.

Since the flux is varying sinusoidally, the rms value of induced emf is obtained by multiplying the average value by the form factor .

Therefore, rms value of emf induced per turns = $1.11 \times 4f \times \phi_m$

= $4.44f\phi_m$ volts

The rms value of induced emf in the entire primary winding = (induced emf per turn) × number of primary turns
\[ i.e, E_1 = 4.44f \phi_m \times N_1 = 4.44f B_m \times A \times N_1 \]

Similarly;

\[ E_2 = 4.44f \phi_m \times N_2 = 4.44f B_m \times A \times N_2 \]

**Transformation Ratio:**

(1) Voltage Transformation Ratio
(2) Current Transformation Ratio

**Voltage Transformation Ratio:**

Voltage transformation ratio can be defined as the ratio of the secondary voltage to the primary voltage denoted by \( K \)

Mathematically given as \( K = \frac{\text{Secondary Voltage}}{\text{Primary Voltage}} = \frac{V_2}{V_1} \)

\[
K = \frac{E_2}{E_1} = \frac{4.44f \phi_m N_2}{4.44f \phi_m N_1} = \frac{N_2}{N_1}
\]

\[
K = \frac{V_2}{V_1} = \frac{E_2}{E_1} = \frac{N_2}{N_1}
\]

**Current Transformation Ratio:**

Consider an ideal transformer and we have the input voltampere is equal to output voltampere.

Mathematically, \( \text{Input Voltampere} = \text{Output Voltampere} \)

\[ V_1 I_1 = V_2 I_2 \]

\[
\frac{V_2}{V_1} = \frac{I_1}{I_2} = K
\]

\[
\therefore \quad K = \frac{V_2}{V_1} = \frac{E_2}{E_1} = \frac{N_2}{N_1} = \frac{I_1}{I_2}
\]
Coupled circuits

- When two coils separated by each other, a change in current in one coil will effect the voltage in another coil by mutual induction.
- Self Inductance: A coil capable of inducing an emf in itself by changing current flowing through it, this property of coil is known as self inductance.
- The self induced emf is directly proportional to the rate of change of current.

\[ e = L \frac{di}{dt} \]

- Where \( L \) = coefficient of self inductance.

Mutual Inductance

- Current in one coil changes, there occurs a change in flux linking with other as result an emf is induced in the adjacent coils.
- The mutually induced emf \( e_2 \) in the second coil is dependent on the rate of change of current in the first coil.

\[ e_2 = M \frac{di_1}{dt} \]

COEFFICIENT OF COUPLING

- \( K = \frac{M}{L_1 L_2} \)
- The two coils is said to be tightly or perfectly coupled only when \( K = 1 \) and therefore \( M = L_1 L_2 \) ’s said to be maximum mutual inductance
- When the distance between the two coils is greater than the coils are said to be loosely packed
- Coefficient of coupling will help in deciding whether the coils are closely packed or loosely packed.
Derivation for Co-efficient of coupling

Dot Convention

- A current entering the dotted terminal of one coil produces an open-circuit voltage which is positively sensed at the dotted terminal of the second coil.
- A current entering the undotted terminal of one coil produces an open-circuit voltage which is positively sensed at the undotted terminal of the second coil.
- The advantage of dot convention is to find out the direction of the winding and direction of flux linking the coil.
- The direction of the flux due to rate of change of flux can be analyzed by right hand thumb rule.

Different connections of coupled circuits

- Series Aiding:
- Series Opposing:
- Parallel Aiding:
- Parallel Opposing
- Refer Circuit diagram and derivation for the class notes.
Equilibrium Equations

- The coil where electrical energy is fed is considered as Primary
- The coil where load is connected to draw the current from mutual induction is Secondary
- There are Two main part in Transformer 1) Core  2) Windings
- Core: The top and bottom part of the core is Yoke, The side limbs are considered as Legs. The core is made up of Silicon steel to avoid the Eddy current and Hysteresis Loss.
- Windings: Basically it is made up of Copper and depends on the current value based on this it is of two types Low Voltage and High Voltage Winding.
There are Two main part in Transformer

1) Core 2) Windings

- Core: The top and bottom part of the core is Yoke, the Vertical portions are considered as of Limbs Legs.
- The core is made up of Silicon steel laminations of thickness 0.33m (CRGO) to avoid the Eddy current and Hysteresis Loss.
- Each laminations are varnished one another and bolted to form a L or T or I shaped structures.
- Windings: Basically it is made up of Copper and depends on the current value based on this it is of two types Low Voltage and High Voltage Winding.
- The LV and HV coils should be placed close to each other as to increase the mutual induction.

- The two coils are separated by insulated materials such as paper, cloth or mica
- Coils maybe placed Helically(Cylindrical) or Sandwiched in the window of transformer
- Rectangular core, two limbs.
- Winding encircles core and Low voltage coil is placed near the limb and insulation by paper and High voltage on it.
- Windings are distributive type and natural cooling is effective and top laminations can be removed for maintenance work.
- Core encircles most of windings
- Natural cooling is not possible
- Maintenance work is difficult
- For HV Transformers
- $1-\Phi$ requires three limbs
- Double magnetic circuit
- Core consists of three limbs, top and bottom yokes.
- Each limb consists of primary and secondary winding (LV and HV winding)
- Three phase transformer can also designed by arranging three single phase transformer in series.
- Shell type (five limb) is used for large transformer because they can be made with a reduced height.
- The cost of three phase shell type transformer is more.
- For cooling of transformer fans are fixed at the radiators.
Core type

- Winding encircles core
- Cylindrical coils
- Natural cooling is effective
- Maintenance work is easy
- Single magnetic circuit
- Low Voltage and distribution type
- Two limbs for 1-phase and three for 3-phase

Shell type

- Core encircles windings
- Disc type

- Natural cooling is not effective
- Maintenance work is difficult
 Double magnetic circuit
 High Voltage transformer
 Three limbs for 1-phase and 6-limbs for three phase

Types of Transformer

 Power Transformer
 Distribution Transformer
 Constant Voltage Transformer
 Constant Current Transformer
 Variable Frequency Transformer
 Auto Transformer

Power transformer of rating 500 mVA 11kv/230v

 Transformer having rating more than 200kva is power transformers
 Usually this transformers are placed near the generating and substations to either step up or step down voltage levels
 The transformers which are used to transform the transmission voltage to the voltage level of primary feeders are called substation transformers
Fig: Power Transformer
Pad mounted & pole mounted distribution transformer

- It changes feeder voltage to the utilization voltage for customer requirements.
- This transformers operate throughout the day therefore iron loss will be throughout the day and copper loss occur only when it is loaded.
- These are low load high efficiency machines.
- It is designed in such way to maintain the small leakage reactance to get good voltage regulation as it want to operate throughout the day.
- Depending on the installation it is of pole mounted or pad mounted as shown in the diagram.
Constant voltage transformer and its output

- It uses the leakage inductance of its secondary windings in combination with external capacitors to create one or more resonant circuits.
- It consists of linear inductor which is unsaturated and this will be primary.
- The non linear inductor (saturated) forms the secondary of the transformer.
- The capacitor connected in parallel saturates by drawing the secondary current due to saturation a constant output voltage is produced.
- Since the output is a quasi sine wave because of the constant in output voltage and this is improved by the compensating winding.
**Constant Current transformer**

- It consists of Primary and secondary winding but one is movable and mounted on the same core.
- A counter weight is used to balance the moving winding.
- The principle is production of two oppositely directed magnetic field.
- If load impedance decreases load current increases due to this large opposition between two magnetic fields produced by primary and secondary.
- Due to repulsion movable winding moves up and further gets separated from stationary and large leakage flux reduces and in turn mutual flux reduces thus secondary voltage reduces.
Fig: Constant Current Transformer
Variable frequency transformer

- The variable frequency transformer (VFT) is essentially a continuously variable phase shifting transformer that can operate at an adjustable phase angle.
- A **variable frequency transformer** is used to transmit electricity between two asynchronous alternating current domains.
- A variable frequency transformer is a doubly-fed electric machine resembling a vertical shaft hydroelectric generator with a three-phase wound rotor, connected by slip rings to one external power circuit. A direct-current torque motor is mounted on the same shaft.
- The phase shift between input and output voltage should also be small over the range of frequencies.
- The applications of VFT are Electronic circuits, Communication, Control and measurement which uses wide band of frequencies.

Auto transformer

- Transformer having only one winding such that part of winding common to both primary and secondary.
- In the fig 1 the auto transformer is step down because N1>N2 and here N1 is common to both sides.
- In the fig 2 the autotransformer is step down because N1<N2 and here N2 is common to both.
- It works on the principle of both induction and conduction.
- Power transfer also takes by both induction and conduction.
- Weight of copper in autotransformer can be reduced.
Fig 1

- Transformer having only one winding such that part of winding common to both primary and secondary
- In the fig 1 the auto transformer is step down because N1> N2 and here N1 is common to both sides
- In the fig 2 the autotransformer is step down because N1< N2 and here N2 is common to both
- It works on the principle of both induction and conduction
 Power transfer also takes by both induction and conduction.
 Weight of copper in autotransformer can be reduced

Advantages of Autotransformer

 Copper required is very less and hence copper loss is reduced.
 Efficiency is higher compared to two winding transformer
 The power rating is more compared to two winding transformer
 The size and cost is less compared to two winding transformer

Applications of Autotransformer

 It is used as variac for starting of machines like Induction machines, Synchronous machines.
 The voltage drop is compensated and acts as booster.
 It used as furnace transformer at the required supply.
 It can be connected between two systems operating at same voltage level.
UNIT- 2 & 3


Losses in Transformer:

Losses of transformer are divided mainly into two types:

1. Iron Loss
2. Copper Losses

Iron Loss:

This is the power loss that occurs in the iron part. This loss is due to the alternating frequency of the emf. Iron loss in further classified into two other losses.

a) Eddy current loss  
b) Hysteresis loss

a) **EDDY CURRENT LOSS:** This power loss is due to the alternating flux linking the core, which will induced an emf in the core called the eddy emf, due to which a current called the eddy current is being circulated in the core. As there is some resistance in the core with this eddy current circulation converts into heat called the eddy current power loss. Eddy current loss is proportional to the square of the supply frequency.

b) **HYSTERESIS LOSS:** This is the loss in the iron core, due to the magnetic reversal of the flux in the core, which results in the form of heat in the core. This loss is directly proportional to the supply frequency.

Eddy current loss can be minimized by using the core made of thin sheets of silicon steel material, and each lamination is coated with varnish insulation to suppress the path of the eddy currents.
Hysterisis loss can be minimized by using the core material having high permeability.

**Copper Loss:**
This is the power loss that occurs in the primary and secondary coils when the transformer is on load. This power is wasted in the form of heat due to the resistance of the coils. This loss is proportional to the sequence of the load hence it is called the Variable loss where as the Iron loss is called as the Constant loss as the supply voltage and frequency are constants.

**Efficiency:**
It is the ratio of the output power to the input power of a transformer

\[
\text{Input} = \text{Output} + \text{Total losses} \\
= \text{Output} + \text{Iron loss} + \text{Copper loss}
\]

Efficiency =

\[
\eta = \frac{\text{output power}}{\text{output power} + \text{Ironloss} + \text{copperloss}}
\]

\[
= \frac{V_2I_2 \cos \phi}{V_2I_2 \cos \phi + W_{iron} + W_{copper}}
\]

Where, \(V_2\) is the secondary (output) voltage, \(I_2\) is the secondary (output) current and \(\cos \Phi\) is the power factor of the load.

The transformers are normally specified with their ratings as KVA,

Therefore,

\[
\text{Efficiency}; \eta = \frac{(KVA) \times 10^3 \times \cos \phi}{(KVA) \times 10^3 \times \cos \phi \times W_{iron} + W_{copper}}
\]

Since the copper loss varies as the square of the load the efficiency of the transformer at any desired load \(n\) is given by

\[
\text{Efficiency}; \eta = \frac{n \times (KVA) \times 10^3 \times \cos \phi}{n \times (KVA) \times 10^3 \times \cos \phi \times W_{iron} + n^2 \times W_{copper}}
\]
where $W_{\text{copper}}$ is the copper loss at full load

$$W_{\text{copper}} = I^2R \text{ watts}$$

**CONDITION FOR MAXIMUM EFFICIENCY:**

In general for the efficiency to be maximum for any device the losses must be minimum. Between the iron and copper losses the iron loss is the fixed loss and the copper loss is the variable loss. When these two losses are equal and also minimum the efficiency will be maximum.

Therefore the condition for maximum efficiency in a transformer is

$$\text{Copper loss} = \text{Iron loss}$$

(whichever is minimum)

**VOLTAGE REGULATION:**

The voltage regulation of a transformer is defined as the change in the secondary terminal voltage between no load and full load at a specified power factor expressed as a percentage of the full load terminal voltage.

$$\% \text{Voltage Regulation} = \frac{(\text{no load Sec. Voltage}) - (\text{full load Sec. Voltage})}{\text{full load Sec. Voltage}} \times 100$$

Voltage regulation is a measure of the change in the terminal voltage of a transformer between No load and Full load. A good transformer has least value of the regulation of the order of $\pm 5\%$

If a load is connected to the secondary, an electric current will flow in the secondary winding and electrical energy will be transferred from the primary circuit through the transformer to the load. In an ideal transformer, the induced voltage in the secondary winding ($V_s$) is in proportion to the primary voltage ($V_p$), and is given by the ratio of the number of turns in the secondary ($N_s$) to the number of turns in the primary ($N_p$) as follows:
Earlier it is seen that a voltage is induced in a coil when the flux linkage associated with the same changed. If one can generate a time varying magnetic field any coil placed in the field of influence linking the same experiences an induced emf. A time varying field can be created by passing an alternating current through an electric coil. This is called mutual induction. The medium can even be air. Such an arrangement is called air cored transformer.

Indeed such arrangements are used in very high frequency transformers. Even though the principle of transformer action is not changed, the medium has considerable influence on the working of such devices. These effects can be summarized as the followings.

1. The magnetizing current required to establish the field is very large, as the reluctance of the medium is very high.

2. There is linear relationship between the mmf created and the flux produced.

3. The medium is non-lossy and hence no power is wasted in the medium.

4. Substantial amount of leakage flux exists.

5. It is very hard to direct the flux lines as we desire, as the whole medium is homogeneous.

If the secondary is not loaded the energy stored in the magnetic field finds its way back to the source as the flux collapses. If the secondary winding is connected to a load then part of the power from the source is delivered to the load through the magnetic field as a link.

The medium does not absorb and lose any energy. Power is required to create the field and not to maintain the same. As the winding losses can be made very small by proper choice of material, the ideal efficiency of a transformer approaches 100%. The large magnetizing current requirement is a major deterrent.
Transformers And Induction Machines

(a)

Primary

Leakage flux

Secondary

Mutual flux
1. Due to the large value for the permeance ($\mu r$ of the order of 1000 as compared to air) the magnetizing current requirement decreases dramatically. This can also be visualized as a dramatic increase in the flux produced for a given value of magnetizing current.

2. The magnetic medium is linear for low values of induction and exhibits saturation type of non-linearity at higher flux densities.

3. The iron also has hysteresis type of non-linearity due to which certain amount of power is lost in the iron (in the form of hysteresis loss), as the B H characteristic is traversed.

4. Most of the flux lines are confined to iron path and hence the mutual flux is increased very much and leakage flux is greatly reduced.

5. The flux can be easily ‘directed’ as it takes the path through steel which gives great freedom for the designer in physical arrangement of the excitation and output windings.
6. As the medium is made of a conducting material eddy currents are induced in the same and produce losses. These are called ‘eddy current losses’. To minimize the eddy current losses the steel core is required to be in the form of a stack of insulated laminations.

From the above it is seen that the introduction of magnetic core to carry the flux introduced two more losses. Fortunately the losses due to hysteresis and eddy current for the available grades of steel are very small at power frequencies. Also the copper losses in the winding due to magnetization current are reduced to an almost insignificant fraction of the full load losses. Hence steel core is used in power transformers.

In order to have better understanding of the behavior of the transformer, initially certain idealizations are made and the resulting ‘ideal’ transformer is studied. These idealizations are as follows:

1. Magnetic circuit is linear and has infinite permeability. The consequence is that a vanishingly small current is enough to establish the given flux. Hysteresis loss is negligible. As all the flux generated confines itself to the iron, there is no leakage flux.

2. Windings do not have resistance. This means that there are no copper losses, nor there is any ohmic drop in the electric circuit.

In fact the practical transformers are very close to this model and hence no major departure is made in making these assumptions. Fig 11 shows a two winding ideal transformer. The primary winding has $T_1$ turns and is connected to a voltage source of $V_1$ volts. The secondary has $T_2$ turns. Secondary can be connected to load impedance for loading the transformer. The primary and secondary are shown on the same limb and separately for clarity.

As a current $I_0$ amps is passed through the primary winding of $T_1$ turns it sets up an MMF of $I_0 T_1$ ampere which is in turn sets up a flux $\phi$ through the core. Since the reluctance of the iron path given by $R = l/\mu A$ is zero as $\mu \approx 1$, a vanishingly small value of current $I_0$ is enough to setup a flux which is finite. As $I_0$ establishes the field inside the transformer
it is called the magnetizing current of the transformer.

\[
\text{Flux } \phi = \frac{\text{mmf}}{\text{Reluctance}} = \frac{I_0 T_1}{l} = \frac{I_0 T_1 A \mu}{l}.
\]
This current is the result of a sinusoidal voltage \( V \) applied to the primary. As the current through the loop is zero (or vanishingly small), at every instant of time, the sum of the voltages must be zero inside the same. Writing this in terms of instantaneous values we have, \( v_1 - e_1 = 0 \) where \( v_1 \) is the instantaneous value of the applied voltage and \( e_1 \) is the induced emf due to Faraday’s principle. The negative sign is due to the application of the Lenz’s law and shows that it is in the form of a voltage drop. Kirchoff’s law application to the loop will result in the same thing.

This equation results in \( v_1 = e_1 \) or the induced emf must be same in magnitude to the applied voltage at every instant of time. Let \( v_1 = V_{1\text{peak}} \cos \omega t \) where \( V_{1\text{peak}} \) is the peak value and \( \omega = 2\pi f \) t. \( f \) is the frequency of the supply. As \( v_1 = e_1; \ e_1 = \frac{d}{dt} \) but \( e_1 = E_{1\text{peak}} \cos \omega t \), \( E_1 = V_1 \). It can be easily seen that the variation of flux linkages can be obtained as \( \psi_1 = \psi_{1\text{peak}} \sin \omega t \). Here \( \psi_{1\text{peak}} \) is the peak value of the flux linkages of the primary.

Thus the RMS primary induced EMF is

\[
e_1 = \frac{d\psi_1}{dt} = \frac{d(\psi_{1\text{peak}} \sin \omega t)}{dt} = \psi_{1\text{peak}} \omega \cos \omega t \quad \text{or the rms value}
\]

\[
E_1 = \frac{\psi_{1\text{peak}} \omega}{\sqrt{2}} = \frac{2\pi f T_1 \phi_m}{\sqrt{2}} = 4.44f \phi_m T_1 \quad \text{volts}
\]

Here \( \psi_{1\text{peak}} \) is the peak value of the flux linkages of the primary. The same mutual flux links the secondary winding. However the magnitude of the flux linkages will be \( \psi_{2\text{peak}} = T_2 \phi_m \). The induced emf in the secondary can be similarly obtained as

\[
e_2 = \frac{d\psi_2}{dt} = \frac{d(\psi_{2\text{peak}} \sin \omega t)}{dt} = \psi_{2\text{peak}} \omega \cos \omega t \quad \text{or the rms value}
\]

\[
E_2 = \frac{2\pi f T_2 \phi_m}{\sqrt{2}} = 4.44f \phi_m T_2 \quad \text{volts}
\]
Which yields the voltage ratio as \( E_1/E_2 = T_1/T_2 \)

**Transformer at loaded condition.**

So far, an unloaded ideal transformer is considered. If now a load impedance \( Z_L \) is connected across the terminals of the secondary winding a load current flows as marked in Fig. 11(c). This load current produces a demagnetizing mmf and the flux tends to collapse. However this is detected by the primary immediately as both \( E_2 \) and \( E_1 \) tend to collapse.

The current drawn from supply increases up to a point the flux in the core is restored back to its original value. The demagnetizing mmf produced by the secondary is neutralized by additional magnetizing mmf produces by the primary leaving the mmf and flux in the core as in the case of no-load. Thus the transformer operates under constant induced emf mode. Thus

\[
i_1 T_1 - i_2 T_2 = i_0 T_1 \quad \text{but} \quad i_0 \to 0
\]

\[
i_2 T_2 = i_1 T_1 \quad \text{and the rms value} \quad I_2 T_2 = I_1 T_1.
\]

If the reference directions for the two currents are chosen as in the Fig. 12, then the above equation can be written in phasor form as,

\[
I_1 T_1 = I_2 T_2 \quad \text{or} \quad I_1 = \frac{T_2}{T_1} I_2
\]

Also

\[
\frac{E_1}{E_2} = \frac{T_1}{T_2} = \frac{I_2}{I_1} \quad E_1 I_1 = E_2 I_2
\]

Thus voltage and current transformation ratio are inverse of one another. If an impedance of \( Z_L \) is connected across the secondary,

\[
Z_i = \frac{E_1}{I_1} = \left(\frac{T_1}{T_2}\right)^2 \cdot \frac{E_2}{I_2} = \left(\frac{T_1}{T_2}\right)^2 \cdot \bar{Z}_L
\]
An impedance of $Z_L$ when viewed ‘through’ a transformer of turns ratio $(T_1/T_2)$ is seen as $(T_1/T_2)^2 Z_L$. Transformer thus acts as an impedance converter. The transformer can be interposed in between a source and a load to ‘match’ the impedance.

![Figure 13: Phasor diagram of Operation of an Ideal Transformer](image)

Finally, the phasor diagram for the operation of the ideal transformer is shown in Fig. 13 in which $\theta_1$ and $\theta_2$ are power factor angles on the primary and secondary sides. As the transformer itself does not absorb any active or reactive power it is easy to see that $\theta_1 = \theta_2$.

Thus, from the study of the ideal transformer it is seen that the transformer provides electrical isolation between two coupled electric circuits while maintaining power invariance at its two ends. However, grounding of loads and one terminal of the transformer on the secondary/primary side are followed with the provision of leakage current detection devices to safe guard the persons working with the devices. Even though the isolation aspect is a desirable one its utility cannot be over emphasized. It can be used to step up or step down the voltage/current at constant volt-ampere. Also, the transformer can be used for impedance matching. In the case of an ideal transformer the efficiency is 100% as there are no losses inside the device.
Practical Transformer

An ideal transformer is useful in understanding the working of a transformer. But it cannot be used for the computation of the performance of a practical transformer due to the non-ideal nature of the practical transformer. In a working transformer the performance aspects like magnetizing current, losses, voltage regulation, efficiency etc are important. Hence the effects of the non-idealization like finite permeability, saturation, hysteresis and winding resistances have to be added to an ideal transformer to make it a practical transformer.

Conversely, if these effects are removed from a working transformer what is left behind is an ideal transformer.

Finite permeability of the magnetic circuit necessitates a finite value of the current to be drawn from the mains to produce the mmf required to establish the necessary flux.

The current and mmf required is proportional to the flux density B that is required to be established in the core.

\[ B = \mu H; \quad B = \frac{\Phi}{A} \]

where A is the area of cross section of the iron core m². H is the magnetizing force which is given by,

\[ H = i \frac{T_1}{l} \]

where l is the length of the magnetic path, m. or

\[ \phi = B.A = \frac{A\mu(iT_1)}{l} = \text{permeance} \times \text{mmf (here that of primary)} \]

The magnetizing force and the current vary linearly with the applied voltage as long as the magnetic circuit is not saturated. Once saturation sets in, the current has to vary in a
nonlinear manner to establish the flux of sinusoidal shape. This non-linear current can be resolved into fundamental and harmonic currents. This is discussed to some extent under harmonics. At present the effect of this non-linear behavior is neglected as a secondary effect. Hence the current drawn from the mains is assumed to be purely sinusoidal and directly proportional to the flux density of operation. This current can be represented by a current drawn by an inductive reactance in the circuit as the net energy associated with the same over a cycle is zero. The energy absorbed when the current increases are returned to the electric circuit when the current collapses to zero. This current is called the magnetizing current of the transformer. The magnetizing current Im is given by Im = E1/Xm where Xm is called the magnetizing reactance. The magnetic circuit being lossy absorbs and dissipates the power depending upon the flux density of operation. These losses arise out of hysteresis, eddy current inside the magnetic core. These are given by the following expressions:

\[
P_h \propto B^{1.6} f \\
P_e \propto B^2 f^2 t^2
\]

\(P_h\) - Hysteresis loss, Watts

\(B\) - Flux density of operation Tesla.

\(f\) - Frequency of operation, Hz

\(t\) - Thickness of the laminations of the core, m.

For a constant voltage, constant frequency operation \(B\) is constant and so are these losses. An active power consumption by the no-load current can be represented in the input circuit as a resistance \(R_c\) connected in parallel to the magnetizing reactance \(X_m\). Thus the no-load current \(I_0\) may be made up of \(I_c\) (loss component) and \(I_m\) (magnetizing component
as) $I_0 = I_c - jI_m^2 cR_c$ - gives the total core losses (i.e. hysteresis + eddy current loss)

$I_m^2 X_m$ - Reactive volt amperes consumed for establishing the mutual flux.

Finite $\mu$ of the magnetic core makes a few lines of flux take to a path through the air. Thus these flux lines do not link the secondary winding. It is called as leakage flux. As the path of the leakage flux is mainly through the air the flux produced varies linearly with the primary current $I_1$. Even a large value of the current produces a small value of flux. This flux produces a voltage drop opposing its cause, which is the current $I_1$. Thus this effect of the finite permeability of the magnetic core can be represented as a series inductive element $jx_1$. This is termed as the reactance due to the primary leakage flux. As this leakage flux varies linearly with $I_1$, the flux linkages per ampere and the primary leakage inductance are constant (This is normally represented by $l_1$ Henry). The primary leakage reactance therefore becomes $x_1 = 2\pi f l_1 \text{ohm}$.

A similar effect takes place on the secondary side when the transformer is loaded. The secondary leakage reactance $jx_2$ arising out of the secondary leakage inductance $l_2$ is given by $x_2 = 2\pi f l_2$. Finally, the primary and secondary windings are wound with copper (sometimes aluminum in small transformers) conductors; thus the windings have a finite resistance (though small). This is represented as a series circuit element, as the power lost and the drop produced in the primary and secondary are proportional to the respective currents. These are represented by $r_1$ and $r_2$ respectively on primary and secondary side. A practical transformers ans these imperfections (taken out and represented explicitly in the electric circuits) is an ideal transformer of turns ratio $T_1 : T_2$ (voltage ratio $E_1 : E_2$). This is seen in Fig. 14. $I_1'$ in the circuit represents the primary current component that is required to flow from the mains in the primary $T_1$ turns to neutralize the demagnetizing secondary current $I_2$ due to the load in the secondary turns. The total primary current
(a) Physical arrangement
By solving this circuit for any load impedance $Z_L$ one can find out the performance of the loaded transformer.

The circuit shown in Fig. 14(b). However, it is not very convenient for use due to the presence of the ideal transformer of turns ratio $T_1 : T_2$. If the turns ratio could be made unity by some transformation the circuit becomes very simple to use. This is done here by replacing the secondary by a ‘hypothetical’ secondary having $T_1$ turns which is ‘equivalent’ to the physical secondary. The equivalence implies that the ampere turns, active and reactive power associated with both the circuits must be the same. Then there is no change as far as their effect on the primary is considered.

**Figure 14: A Practical Transformer**

$$\vec{I}_1 = \vec{I}_2 + \vec{I}_0$$

Here $I'_2 T_1 = I_2 T_2$ or $I'_2 = I_2 \frac{T_2}{T_1}$

Thus $I_1 = I_2 \frac{T_2}{T_1} + I_0$
Thus

\[
V_2' = aV_2, \quad I_2' = \frac{I_2}{a}, \quad r_2' = a^2r_2, \quad x_{l2}' = a^2x_{l2} \quad Z_L' = a^2Z_L.
\]

where a -turns ratio \(T_1/T_2\)

As the ideal transformer in this case has a turns ratio of unity the potentials on either side are the same and hence they may be conductively connected dispensing away with the ideal transformer. This particular equivalent circuit is as seen from the primary side. It is also possible to refer all the primary parameters to secondary by making the hypothetical equivalent primary winding on the input side having the number of turns to be \(T_2\). Such an equivalent circuit having all the parameters referred to the secondary side is shown in fig.

The equivalent circuit can be derived, with equal ease, analytically using the Kirchoff’s equations applied to the primary and secondary. Referring to fig. 14(a), we have (by neglecting the shunt branch)

\[
\begin{align*}
V_1 & = E_1 + I_1(r_1 + jx_{i1}) \\
E_2 & = V_2 + I_2(r_2 + jx_{i2}) \\
T_1 I_0 & = T_1 I_1 + T_2 I_2 \quad \text{or} \quad I_1 = -\frac{I_2}{a} + I_0 \\
& = \frac{I_2}{a} + I_e + I_m \\
\alpha & = \frac{T_1}{T_2}.
\end{align*}
\]

Multiply both sides of Eqn.34 by ‘a’ [This makes the turns ratio unity and retains the power invariance].

\[
aE_2 = aV_2 + aI_2(r_2 + jx_{i2}) \quad \text{but} \quad aE_2 = E_1
\]
Substituting in Eqn we have

\[
V_1 = aV_2 + aI_2(r_2 + jx_{l2}) + I_1(r_1 + jx_{l1}) \\
= V'_2 + I_1(a^2r_2 + ja^2x_{l2}) + I_1(r_1 + jx_{l1}) \\
= V'_2 + I_1(r_1 + r'_2 + jx_{l1} + x'_{l2})
\]

A similar procedure can be used to refer all parameters to secondary side. (Shown in fig)
Figure 15: Equivalent Circuit Referred to the Secondary Side
Phasor Diagrams

The resulting equivalent circuit as shown in Fig. 16 is known as the exact equivalent circuit. This circuit can be used for the analysis of the behavior of the transformers. As the no-load current is less than 1% of the load current a simplified circuit known as ‘approximate’ equivalent circuit (see Fig. 16(b)) is usually used, which may be further simplified to the one shown in Fig. 16(c).

On similar lines to the ideal transformer the phasor diagram of operation can be drawn for a practical transformer also. The positions of the current and induced emf phasor are not known uniquely if we start from the phasor $V_1$. Hence it is assumed that the phasor
is known. The $E_1$ and $E_2$ phasor are then uniquely known. Now, the magnetizing and loss components of the currents can be easily represented. Once $I_0$ is known, the drop that takes place in the primary resistance and series reactance can be obtained which when added to $E_1$ gives uniquely the position of $V_1$ which satisfies all other parameters. This is represented in Fig. 17(a) as phasor diagram on no-load.

Next we proceed to draw the phasor diagram corresponding to a loaded transformer. The position of the $E_2$ vector is known from the flux phasor. Magnitude of $I_2$ and the load power factor angle $\theta_2$ are assumed to be known. But the angle $\theta_2$ is defined with respect to the terminal voltage $V_2$ and not $E_2$. By trial and error the position of $I_2$ and $V_2$ are determined. $V_2$ should also satisfy the Kirchoff’s equation for the secondary. Rest of the construction of the phasor diagram then becomes routine. The equivalent primary current $I_2'$ is added vectorially to $I_0$ to yield $I_1$. $I_1(r_1+jx_1)$ is added to $E_1$ to yield $V_1$. This is shown in fig. 17(b) as phasor diagram for a loaded transformer.
Testing of Transformers

The structure of the circuit equivalent of a practical transformer is developed earlier. The performance parameters of interest can be obtained by solving that circuit for any load conditions. The equivalent circuit parameters are available to the designer of the transformers from the various expressions that he uses for designing the transformers. But for a user these are not available most of the times. Also when a transformer is rewound with different primary and secondary windings the equivalent circuit also changes. In order to get the equivalent circuit parameters test methods are heavily depended upon. From the analysis of the equivalent circuit one can determine the electrical parameters. But if the temperature rise of the transformer is required, then test method is the most dependable one. There are several tests that can be done on the transformer; however a few common ones are discussed here.
Winding resistance test

This is nothing but the resistance measurement of the windings by applying a small d.c voltage to the winding and measuring the current through the same. The ratio gives the winding resistance, more commonly feasible with high voltage windings. For low voltage windings a resistance-bridge method can be used. From the d.c resistance one can get the a.c. resistance by applying skin effect corrections.

![Figure 18: Polarity Test](image)

Polarity Test

This is needed for identifying the primary and secondary phasor polarities. It is a must for poly phase connections. Both a.c. and d.c methods can be used for detecting the polarities of the induced emfs. The dot method discussed earlier is used to indicate the polarities. The transformer is connected to a low voltage a.c. source with the connections made as shown in the fig. 18(a). A supply voltage $V_s$ is applied to the primary and the readings of the voltmeters $V_1$, $V_2$ and $V_3$ are noted. $V_1 : V_2$ gives the turns ratio. If $V_3$ reads $V_1 - V_2$ then assumed dot locations are correct (for the connection shown). The
beginning and end of the primary and secondary may then be marked by A1 –A2 and a1 –a2 respectively.

If the voltage rises from A1 to A2 in the primary, at any instant it does so from a1 to a2 in the secondary. If more secondary terminals are present due to taps taken from the windings they can be labeled as a3, a4, a5, a6. It is the voltage rising from smaller number towards larger ones in each winding. The same thing holds good if more secondaries are present.

Fig. 18(b) shows the d.c. method of testing the polarity. When the switch S is closed if the secondary voltage shows a positive reading, with a moving coil meter, the assumed polarity is correct. If the meter kicks back the assumed polarity is wrong.

**Open Circuit Test**

![Physical Arrangement](image1)

![Equivalent Circuit](image2)

Figure 19: No Load Test
As the name suggests, the secondary is kept open circuited and nominal value of the input voltage is applied to the primary winding and the input current and power are measured. In Fig. 19(a) V,A,W are the voltmeter, ammeter and wattmeter respectively. Let these meters read V1, I0 and W0 respectively. Fig. 19(b) shows the equivalent circuit of the transformer under this test. The no load current at rated voltage is less than 1 percent of nominal current and hence the loss and drop that take place in primary impedance r1 +jx11 due to the no load current I0 is negligible. The active component Ic of the no load current I0 represents the core losses and reactive current Im is the current needed for the magnetization.

Thus the watt meter reading

\[
W_0 = V_1 I_c = P_{core}
\]

\[
I_c = \frac{W_0}{V_1}
\]

\[
I_m = \sqrt{I_0^2 - I_c^2} \quad \text{or}
\]

\[
R_c = \frac{V_1}{I_c} \quad \text{and} \quad X_m = \frac{V_1}{I_m}
\]
The parameters measured already are in terms of the primary. Sometimes the primary voltage required may be in kilo-Volts and it may not be feasible to apply nominal voltage to primary from the point of safety to personnel and equipment. If the secondary voltage is low, one can perform the test with LV side energized keeping the HV side open circuited. In this case the parameters that are obtained are in terms of LV. These have to be referred to HV side if we need the equivalent circuit referred to HV side.

Sometimes the nominal value of high voltage itself may not be known, or in doubt, especially in a rewound transformer. In such cases an open circuit characteristics is first obtained, which is a graph showing the applied voltage as a function of the no load current.

This is a non linear curve as shown in Fig. 20. This graph is obtained by noting the current drawn by transformer at different applied voltage, keeping the secondary open circuited. The usual operating point selected for operation lies at some standard voltage around the knee point of the characteristic. After this value is chosen as the nominal value the parameters are calculated as mentioned above.
Short Circuit Test

The purpose of this test is to determine the series branch parameters of the equivalent circuit of Fig. 21(b). As the name suggests, in this test primary applied voltage, the current and power input are measured keeping the secondary terminals short circuited. Let these values be $V_{sc}$, $I_{sc}$ and $W_{sc}$ respectively. The supply voltage required to circulate rated current through the transformer is usually very small and is of the order of a few percent of the nominal voltage. The excitation current which is only 1 percent or less even at rated voltage becomes negligibly small during this test and hence is neglected. The shunt branch is thus assumed to be absent. Also $I_1 = I_2$ as $I_0 \approx 0$. Therefore $W_{sc}$ is the sum of the copper losses in primary and secondary put together. The reactive power consumed is that absorbed by the leakage reactance of the two windings.

\[
W_{sc} = I_{sc}^2 (r_1 + r_2') \\
Z_{sc} = \frac{V_{sc}}{I_{sc}} \\
(x_{l1} + x_{l2}') = \sqrt{Z_{sc}^2 - (r_1 + r_2')^2}
\]
If the approximate equivalent circuit is required then there is no need to separate $r'1$ and $r'2$ or $x'11$ and $x'12$. However if the exact equivalent circuit is needed then either $r'1$ or $r'2$ is determined from the resistance measurement and the other separated from the total.

As for the separation of $x11$ and $x'12$ is concerned, they are assumed to be equal. This is a fairly valid assumption for many types of transformer windings as the leakage flux paths are through air and are similar.
Load Test

Load Test helps to determine the total loss that takes place, when the transformer is loaded. Unlike the tests described previously, in the present case nominal voltage is applied across the primary and rated current is drawn from the secondary. Load test is used mainly

1. to determine the rated load of the machine and the temperature rise
2. to determine the voltage regulation and efficiency of the transformer.

Rated load is determined by loading the transformer on a continuous basis and observing the steady state temperature rise. The losses that are generated inside the transformer on load appear as heat. This heats the transformer and the temperature of the transformer increases. The insulation of the transformer is the one to get affected by this rise in the temperature. Both paper and oil which are used for insulation in the transformer start getting degenerated and get decomposed. If the flash point of the oil is reached the transformer goes up in flames. Hence to have a reasonable life expectancy the loading of the transformer must be limited to that value which gives the maximum temperature rise tolerated by the insulation. This aspect of temperature rise cannot be guessed from the electrical equivalent circuit. Further, the losses like dielectric losses and stray load losses are not modeled in the equivalent circuit and the actual loss under load condition will be in error to that extent.

Many external means of removal of heat from the transformer in the form of different cooling methods give rise to different values for temperature rise of insulation. Hence these permit different levels of loading for the same transformer. Hence the only sure way of ascertaining the rating is by conducting a load test. It is rather easy to load a transformer of small ratings. As the rating increases it becomes difficult to find a load that can absorb the requisite power and a source to feed the necessary current. As the transformers come in varied transformation ratios, in many cases it becomes extremely difficult to get suitable load impedance.
Further, the temperature rise of the transformer is due to the losses that take place ‘inside’ the transformer. The efficiency of the transformer is above 99% even in modest sizes which means 1 percent of power handled by the transformer actually goes to heat up the machine. The remaining 99% of the power has to be dissipated in a load impedance external to the machine. This is very wasteful in terms of energy also. (If the load is of unity power factor) Thus the actual loading of the transformer is seldom resorted to. Equivalent loss methods of loading and ‘Phantom’ loading are commonly used in the case of transformers.

The load is applied and held constant till the temperature rise of transformer reaches a steady value. If the final steady temperature rise is lower than the maximum permissible value, then load can be increased else it is decreased. That load current which gives the maximum permissible temperature rise is declared as the nominal or rated load current and the volt amperes are computed using the same.

In the equivalent loss method a short circuit test is done on the transformer. The short circuit current is so chosen that the resulting loss taking place inside the transformer is equivalent to the sum of the iron losses, full load copper losses and assumed stray load losses. By this method even though one can pump in equivalent loss inside the transformer, the actual distribution of this loss vastly differs from that taking place in reality. Therefore this test comes close to a load test but does not replace one.
In Phantom loading method two identical transformers are needed. The windings are connected back to back as shown in Fig. 22. Suitable voltage is injected into the loop formed by the two secondaries such that full load current passes through them. An equivalent current then passes through the primary also. The voltage source V1 supplies the magnetizing current and core losses for the two transformers. The second source supplies the load component of the current and losses due to the same. There is no power wasted in a load (as a matter of fact there is no real load at all) and hence the name Phantom or virtual loading. The power absorbed by the second transformer which acts as a load is
pushed back in to the mains. The two sources put together meet the core and copper losses of the two transformers. The transformers work with full flux drawing full load currents and hence are closest to the actual loading condition with a physical load.

**Voltage Regulation**

Modern power systems operate at some standard voltages. The equipments working on these systems are therefore given input voltages at these standard values, within certain agreed tolerance limits. In many applications this voltage itself may not be good enough for obtaining the best operating condition for the loads. A transformer is interposed in between the load and the supply terminals in such cases. There are additional drops inside the transformer due to the load currents. While input voltage is the responsibility of the supply provider, the voltage at the load is the one which the user has to worry about.

If undue voltage drop is permitted to occur inside the transformer the load voltage becomes too low and affects its performance. It is therefore necessary to quantify the drop that takes place inside a transformer when certain load current, at any power factor, is drawn from its output leads. This drop is termed as the voltage regulation and is expressed as a ratio of the terminal voltage (the absolute value per se is not too important).

The voltage regulation can be defined in two ways - Regulation Down and Regulation up. These two definitions differ only in the reference voltage as can be seen below.**Regulation down:** This is defined as ” the change in terminal voltage when a load current at any power factor is applied, expressed as a fraction of the no-load terminal voltage”.

Expressed in symbolic form we have,

\[ \text{Regulation} = \frac{|V_{nl}| - |V_i|}{|V_i|} \]
Vnl is the no-load terminal voltage. Vl is load voltage. Normally full load regulation is of interest as the part load regulation is going to be lower.

This definition is more commonly used in the case of alternators and power systems as the user-end voltage is guaranteed by the power supply provider. He has to generate proper no-load voltage at the generating station to provide the user the voltage he has asked for. In the expressions for the regulation, only the numerical differences of the voltages are taken and not vector differences.

In the case of transformers both definitions result in more or less the same value for the regulation as the transformer impedance is very low and the power factor of operation is quite high. The power factor of the load is defined with respect to the terminal voltage on load. Hence a convenient starting point is the load voltage. Also the full load output voltage is taken from the name plate. Hence regulation up has some advantage when it comes to its application. Fig. 23 shows the phasor diagram of operation of the transformer under loaded condition. The no-load current I0 is neglected in view of the large magnitude of I’2. Then
Figure 23: Regulation of Transformer.
\[ I_1 = I_2. \]

\[ V_1 = I_2' (R_e + jX_e) + V_2' \]

\[ OD = V_1 = \sqrt{[OA + AB + BC]^2 + [CD]^2} \]

\[ = \sqrt{[V_2' + I_2' R_e \cos \phi + I_2' X_e \sin \phi]^2 + [I_2' X_e \cos \phi - I_2' R_e \sin \phi]^2} \]

\[ \phi - \text{power factor angle}, \]

\[ \theta - \text{internal impedance angle} = \tan^{-1} \frac{X_e}{R_e} \]

Also,

\[ V_1 = V_2' + I_2' (R_e + jX_e) \]

\[ = V_2' + I_2' (\cos \phi - j \sin \phi) (R_e + jX_e) \]

\[ \therefore \text{Regulation} R = \frac{|V_1| - |V_2'|}{|V_2'|} = \sqrt{(1 + v_1)^2 + v_2^2} - 1 \]

\[ (1 + v_1)^2 + v_2^2 \approx (1 + v_1)^2 + v_2^2 \cdot \frac{2(1 + v_1)}{2(1 + v_1)} + \left[ \frac{v_2^2}{2(1 + v_1)} \right]^2 = (1 + v_1 + \frac{v_2^2}{2(1 + v_1)})^2 \]

Taking the square root

\[ \sqrt{(1 + v_1)^2 + v_2^2} = 1 + v_1 + \frac{v_2^2}{2(1 + v_1)} \]

where \( v_1 = e_r \cos \phi + e_x \sin \phi \) and \( v_2 = e_x \cos \phi - e_r \sin \phi \)

\[ e_r = \frac{I_2' R_e}{V_2} = \text{per unit resistance drop} \]

\[ e_x = \frac{I_2' X_e}{V_2} = \text{per unit reactance drop} \]

as \( v_1 \) and \( v_2 \) are small.

\[ \therefore R \approx 1 + v_1 + \frac{v_2^2}{2(1 + e_1)} - 1 \approx v_1 + \frac{v_2^2}{2} \]

\[ \therefore \text{regulation} R = e_r \cos \phi \pm e_x \sin \phi + \frac{(e_x \sin \phi - e_r \cos \phi)^2}{2} \]
Powers higher than 2 for $v_1$ and $v_2$ are negligible as $v_1$ and $v_2$ are already small. As $v_2$ is small its second power may be neglected as a further approximation and the expression for the regulation of the transform boils down to regulation

$$R = e_r \cos \phi \pm e_x \sin \phi$$

The negative sign is applicable when the power factor is leading. It can be seen from the above expression, the full load regulation becomes zero when the power factor is leading

$$e_r \cos \phi = e_x \sin \phi \text{ or } \tan \phi = e_r/e_x$$

or the power factor angle $\phi = \tan^{-1}(e_r/e_x) = \tan^{-1}(R_e/X_e)$ leading.

Similarly, the value of the regulation is maximum at a power factor angle $\phi = \tan^{-1}(e_x/e_r) = \tan^{-1}(X_e/R_e)$ lagging.

An alternative expression for the regulation of a transformer can be derived by the method shown in Fig. 24. Here the phasor are resolved along the current axis and normal to it.

We have,

$$OD^2 = (OA + AB)^2 + (BC + CD)^2 = (V'_2 \cos \phi + I'_2 R_e)^2 + (V'_2 \sin \phi + I'_2 X_e)^2$$

$$\therefore \text{ Regulation } R = \frac{OD - V'_2}{V'_2} = \frac{OD}{V'_2} - 1$$

$$\sqrt{\left(\frac{V'_2 \cos \phi + I'_2 R_e}{V'_2}\right)^2 + \left(\frac{V'_2 \sin \phi + I'_2 X_e}{V'_2}\right)^2} - 1$$

$$= \sqrt{(\cos \phi + R_p u)^2 + (\sin \phi + X_{p.u})^2} - 1$$
Thus this expression may not be as convenient as the earlier one due to the square root involved. Fig. shows the variation of full load regulation of a typical transformer as the power factor is varied from zero power factor leading, through unity power factor, to zero power factor lagging.

It is seen from Fig. that the full load regulation at unity power factor is nothing but the percentage resistance of the transformer. It is therefore very small and negligible. Only with low power factor loads the drop in the series impedance of the transformer contributes substantially to the regulation. In small transformers the designer tends to keep the $X_e$ very low (less than 5%) so that the regulation performance of the transformer is satisfactory.
A low value of the short circuit impedance/reactance results in a large short circuit current in case of a short circuit. This in turn results in large mechanical forces on the winding. So, in large transformers the short circuit impedance is made high to give better short circuit protection to the transformer which results in poorer regulation performance. In the case of transformers provided with taps on windings, so that the turns ratio can be changed, the voltage regulation is not a serious issue. In other cases care has to be exercised in the selection of the short circuit impedance as it affects the voltage regulation.


Efficiency

Transformers which are connected to the power supplies and loads and are in operation are required to handle load current and power as per the requirements of the load. An unloaded transformer draws only the magnetization current on the primary side, the secondary current being zero. As the load is increased the primary and secondary currents increase as per the load requirements. The volt amperes and wattage handled by the transformer also increases. Due to the presence of no load losses and I2R losses in the windings certain amount of electrical energy gets dissipated as heat inside the transformer. This gives rise to the concept of efficiency.

Efficiency of a power equipment is defined at any load as the ratio of the power output to the power input. Putting in the form of an expression,

\[
\text{Efficiency } \eta = \frac{\text{output power}}{\text{input power}} = \frac{\text{Input power} - \text{losses inside the machine}}{\text{Input power}}
\]

\[
= 1 - \frac{\text{losses inside the machine}}{\text{input power}} = 1 - \text{deficiency}
\]

\[
= \frac{\text{output power}}{\text{output} + \text{losses inside the machine}}
\]

More conveniently the efficiency is expressed in percentage. \(\% \eta = \frac{\text{output power}}{\text{input power}} \times 100\)

While the efficiency tells us the fraction of the input power delivered to the load, the deficiency focuses our attention on losses taking place inside transformer. As a matter of fact the losses heat up machine. The temperature rise decides the rating of the equipment.

The temperature rise of the machine is a function of heat generated the structural configuration, method of cooling and type of loading (or duty cycle of load). The peak temperature attained directly affects the life of the insulations of the machine for any class of insulation.
These aspects are briefly mentioned under section 7.5 on load test.

A typical curve for the variation of efficiency as a function of output is given in Fig. The losses that take place inside the machine expressed as a fraction of the input is some times termed as deficiency. Except in the case of an ideal machine, a certain fraction of the input power gets lost inside the machine while handling the power. Thus the value for the efficiency is always less than one. In the case of a.c. machines the rating is expressed in terms of apparent power. It is nothing but the product of the applied voltage and the current drawn. The actual power delivered is a function of the power factor at which this current is drawn. As the reactive power shuttles between the source and the load and has a zero average value over a cycle of the supply wave it does not have any direct effect on the efficiency. The reactive power however increases the current handled by the machine and
the losses resulting from it. Therefore the losses that take place inside a transformer at any
given load play a vital role in determining the efficiency. The losses taking place inside a
transformer can be enumerated as below:

1. Primary copper loss
2. Secondary copper loss
3. Iron loss
4. Dielectric loss
5. Stray load loss

These are explained in sequence below.

Primary and secondary copper losses take place in the respective winding resistances due to
the flow of the current in them.

\[ P_c = I_1^2 r_1 + I_2^2 r_2 = I_2^2 R_e \]

The primary and secondary resistances differ from their d.c. values due to skin
effect and the temperature rise of the windings. While the average temperature rise can be
approximately used, the skin effect is harder to get analytically. The short circuit test gives
the value of Re taking into account the skin effect.

The iron losses contain two components - Hysteresis loss and Eddy current loss. The
Hysteresis loss is a function of the material used for the core.

\[ P_h = K_h B^{1.6} f \]

For constant voltage and constant frequency operation this can be taken to be
constant. The eddy current loss in the core arises because of the induced emf in the steel
lamination sheets and the eddies of current formed due to it. This again produces a power
loss Pe in the lamination.
where \( t \) is the thickness of the steel lamination used. As the lamination thickness is much smaller than the depth of penetration of the field, the eddy current loss can be reduced by reducing the thickness of the lamination. Present day laminations are of 0.25 mm thickness and are capable of operation at 2 Tesla. These reduce the eddy current losses in the core. This loss also remains constant due to constant voltage and frequency of operation. The sum of hysteresis and eddy current losses can be obtained by the open circuit test.

The dielectric losses take place in the insulation of the transformer due to the large electric stress. In the case of low voltage transformers this can be neglected. For constant voltage operation this can be assumed to be a constant.

The stray load losses arise out of the leakage fluxes of the transformer. These leakage fluxes link the metallic structural parts, tank etc. and produce eddy current losses in them. Thus they take place ’all round’ the transformer instead of a definite place, hence the name ’stray’. Also the leakage flux is directly proportional to the load current unlike the mutual flux which is proportional to the applied voltage. Hence this loss is called ’stray load’ loss. This can also be estimated experimentally. It can be modeled by another resistance in the series branch in the equivalent circuit. The stray load losses are very low in air-cored transformers due to the absence of the metallic tank.

Thus, the different losses fall in to two categories Constant losses (mainly voltage dependant) and Variable losses (current dependant). The expression for the efficiency of the transformer operating at a fractional load \( x \) of its rating, at a load power factor of \( \Theta_2 \), can be written as

\[
\eta = \frac{x S \cos \theta_2}{x S \cos \theta_2 + P_{\text{const}} + x^2 P_{\text{var}}}
\]

Here \( S \) in the volt ampere rating of the transformer (\( V'2 \ I'2 \) at full load), \( P_{\text{const}} \) being constant losses and \( P_{\text{var}} \) the variable losses at full load.
For a given power factor an expression for $\Theta_2$ in terms of the variable $x$ is thus obtained. By differentiating $\Theta_2$ with respect to $x$ and equating the same to zero, the condition for maximum efficiency is obtained. In the present case that condition comes out to be

$$P_{\text{const}} = x^2 P_{\text{var}} \text{ or } x = \sqrt{\frac{P_{\text{const}}}{P_{\text{var}}}}$$

That is, when constant losses equal the variable losses at any fractional load $x$ the efficiency reaches a maximum value. The maximum value of that efficiency at any given power factor is given by,

$$\eta_{\text{max}} = \frac{x S \cos \theta_2}{x S \cos \theta_2 + 2P_{\text{const}}} = \frac{x S \cos \theta_2}{x S \cos \theta_2 + 2x^2 P_{\text{var}}}$$

From the expression for the maximum efficiency it can be easily deduced that this maximum value increases with increase in power factor and is zero at zero power factor of the load. It may be considered a good practice to select the operating load point to be at the maximum efficiency point. Thus if a transformer is on full load, for most part of the time then the $\Theta_2$ max can be made to occur at full load by proper selection of constant and variable losses. However, in the modern transformers the iron losses are so low that it is practically impossible to reduce the full load copper losses to that value. Such a design wastes lot of copper.

**All day efficiency**

Large capacity transformers used in power systems are classified broadly into Power transformers and Distribution transformers. The former variety is seen in generating stations and large substations. Distribution transformers are seen at the distribution substations. The basic difference between the two types arise from the fact that the power transformers are switched in or out of the circuit depending upon the load to be handled by them. Thus at 50% load on the station only 50% of the transformers need to be connected in the circuit.
On the other hand a distribution transformer is never switched off. It has to remain in the circuit irrespective of the load connected. In such cases the constant loss of the transformer continues to be dissipated. Hence the concept of energy based efficiency is defined for such transformers. It is called ‘all day’ efficiency. The all day efficiency is thus the ratio of the energy output of the transformer over a day to the corresponding energy input. One day is taken as a duration of time over which the load pattern repeats itself. This assumption, however, is far from being true. The power output varies from zero to full load depending on the requirement of the user and the load losses vary as the square of the fractional loads.

The no-load losses or constant losses occur throughout the 24 hours. Thus, the comparison of loads on different days becomes difficult. Even the load factor, which is given by the ratio of the average load to rated load, does not give satisfactory results. The calculation of the all day efficiency is illustrated below with an example. The graph of load on the transformer, expressed as a fraction of the full load is plotted against time in Fig. 27. In an actual situation the load on the transformer continuously changes. This has been presented by a stepped curve for convenience. The average load can be calculated by
Where $P_i$ is the load during an interval $i$. $n$ intervals are assumed. $x_i$ is the fractional load.

$$S_i = x_i S_n$$

where $S_n$ is nominal load. The average loss during the day is given by

$$\text{Average loss} = P_i + \frac{P_c \sum_{i=1}^{n} x_i^2 t_i}{24}$$

This is a non-linear function. For the same load factor different average loss can be there depending upon the values of $x_i$ and $t_i$. Hence a better option would be to keep the constant losses very low to keep the all day efficiency high. Variable losses are related to load and are associated with revenue earned. The constant losses on the other hand has to be incurred to make the service available. The concept of all day efficiency may therefore be more useful for comparing two transformers subjected to the same load cycle.

The concept of minimizing the lost energy comes into effect right from the time of procurement of the transformer. The constant losses and variable losses are capitalized and added to the material cost of the transformer in order to select the most competitive one which gives minimum cost taking initial cost and running cost put together. Obviously the iron losses are capitalized more in the process to give an effect to the maximization of energy efficiency. If the load cycle is known at this stage, it can also be incorporated in computation of the best transformer.
Harmonics

In addition to the operation of transformers on the sinusoidal supplies, the harmonic behavior becomes important as the size and rating of the transformer increases. The effects of the harmonic currents are

1. Additional copper losses due to harmonic currents
2. Increased core losses
3. Increased electro magnetic interference with communication circuits.

On the other hand the harmonic voltages of the transformer cause

1. Increased dielectric stress on insulation
2. Electro static interference with communication circuits.
3. Resonance between winding reactance and feeder capacitance.

In the present times a greater awareness is generated by the problems of harmonic voltages and currents produced by non-linear loads like the power electronic converters.

These combine with non-linear nature of transformer core and produce severe distortions in voltages and currents and increase the power loss. Thus the study of harmonics is of great practical significance in the operation of transformers. The discussion here is confined to the harmonics generated by transformers only.
**Single phase transformers**

Modern transformers operate at increasing levels of saturation in order to reduce the weight and cost of the core used in the same. Because of this and due to the hysteresis, the transformer core behaves as a highly non-linear element and generates harmonic voltages and currents. This is explained below. Fig. 34 shows the manner in which the shape of the magnetizing current can be obtained and plotted. At any instant of the flux density wave the ampere turns required to establish the same is read out and plotted, traversing the hysteresis loop once per cycle. The sinusoidal flux density curve represents the sinusoidal applied voltage to some other scale. The plot of the magnetizing current which is peaky is analyzed using Fourier analysis. The harmonic current components are obtained from this analysis. These harmonic currents produce harmonic fields in the core and harmonic voltages in the windings. Relatively small value of harmonic fields generates considerable magnitude of harmonic voltages. For example a 10% magnitude of 3rd harmonic flux produces 30% magnitude of 3rd harmonic voltage. These effects get even more pronounced for higher order harmonics. As these harmonic voltages get short circuited through the low impedance
of the supply they produce harmonic currents. These currents produce effects according to Lenz’s law and tend to neutralize the harmonic flux and bring the flux wave to a sinusoid. Normally third harmonic is the largest in its magnitude and hence the discussion is based on it. The same can be told of other harmonics also. In the case of a single phase transformer the harmonics are confined mostly to the primary side as the source impedance is much smaller compared to the load impedance. The understanding of the phenomenon becomes more clear if the transformer is supplied with a sinusoidal current source. In this case current has to be sinusoidal and the harmonic currents cannot be supplied by the source and hence the induced emf will be peaky containing harmonic voltages. When the load is connected on the secondary side the harmonic currents flow through the load and voltage tends to become sinusoidal. The harmonic voltages induce electric stress on dielectrics and increased electro static interference. The harmonic currents produce losses and electromagnetic interference as already noted above.
UNIT- 4


Poly Phase connections and Poly phase Transformers

The individual transformers are connected in a variety of ways in a power system. Due to the advantages of polyphase power during generation, transmission and utilization polyphase power handling is very important. As an engineering application is driven by techno-economic considerations, no single connection or setup is satisfactory for all applications. Thus transformers are deployed in many forms and connections. Star and mesh connections are very commonly used. Apart from these, vee or open delta connections, zigzag connections , T connections, auto transformer connections, multi winding transformers etc. are a few of the many possibilities. A few of the common connections and the technical and economic considerations that govern their usage are discussed here. Literature abounds in the description of many other. Apart from the characteristics and advantages of these, one must also know their limitations and problems, to facilitate proper selection of a configuration for an application.

Many polyphase connections can be formed using single phase transformers. In some cases it may be preferable to design, develop and deploy a polyphase transformer itself. In a balanced two phase system we encounter two voltages that are equal in magnitude differing in phase by 90°. Similarly, in a three phase system there are three equal voltages differing in phase 120 electrical degrees. Further there is an order in which they reach a particular voltage magnitude. This is called the phase sequence. In some applications like a.c. to d.c. conversion, six phases or more may be encountered.
Transformers used in all these applications must be connected properly for proper functioning. The basic relationship between the primary and secondary voltages (brought about by a common mutual flux and the number of turns), the polarity of the induced emf (decided by polarity test and used with dot convention) and some understanding of the magnetic circuit are all necessary for the same. To facilitate the manufacturer and users, international standards are also available. Each winding has two ends designated as 1 and 2. The HV winding is indicated by capital letters and the LV winding by small letters. If more terminals are brought out from a winding by way of taps there are numbered in the increasing numbers in accordance to their distance from 1 (eg A1,A2,A3...). If the induced emf at an instant is from A1 to A2 on the HV winding it will rise from a1 to a2 on the LV winding.

Out of the different polyphase connections three phase connections are mostly encountered due to the wide spread use of three phase systems for generation, transmission and utilization. Three balanced 3-phase voltages can be connected in star or mesh fashion to yield a balanced 3-phase 3-wire system. The transformers that work on the 3-phase supply have star, mesh or zig-zag connected windings on either primary secondary or both. In addition to giving different voltage ratios, they introduce phase shifts between input and output sides. These connections are broadly classified into 4 popular vector groups.

1. Group I: zero phase displacement between the primary and the secondary.

2. Group II: 180° phase displacement.

3. Group III: 30° lag phase displacement of the secondary with respect to the primary.

4. Group IV: 30° lead phase displacement of the secondary with respect to the primary.

A few examples of the physical connections and phasor diagrams are shown in Fig. 35 and Fig. 36 corresponding to each group. The capital letters indicates primary and the small letters the secondary. D/d stand for mesh, Y/y - for star, Z/z for zig-zag. The angular displacement of secondary with respect to the primary are shown as clock position, 0°.
Group 1 $0^\circ$ Phase shift

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(a)
Group 3 30° Phase shift

W indings & Terminals

E.M.F. Vector diagrams

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Dept. Of EEE, SJBIT
referring to 12 o’clock position. These vector groups are especially important when two or more transformers are to be connected in parallel.

Star connection is normally cheaper as there are fewer turns and lesser cost of insulation. The advantage becomes more with increase in voltage above 11kv. In a star connected winding with earthed-neutral the maximum voltage to the earth is \((1\sqrt{3})\) of the line voltage.

Also star connection permits mixed loading due to the presence of the neutral. Mesh connections are advantageous in low voltage transformers as insulation costs are insignificant and the conductor size becomes \((1\sqrt{3})\) of that of star connection and permits ease of winding. The common polyphase connections are briefly discussed now.
Star/star (Yy0, Yy6) connection This is the most economical one for small high voltage transformers. Insulation cost is highly reduced. Neutral wire can permit mixed loading. Triplen harmonics are absent in the lines. These triplen harmonic currents cannot flow, unless there is a neutral wire. This connection produces oscillating neutral. Three phase shell type units have large triplen harmonic phase voltage. However three phase core type transformers work satisfactorily. A tertiary mesh connected winding may be required to stabilize the oscillating neutral due to third harmonics in three phase banks.

Mesh/mesh (Dd0, Dd6) This is an economical configuration for large low voltage transformers. Large amount of unbalanced load can be met with ease. Mesh permits a circulating path for triplen harmonics thus attenuates the same. It is possible to operate with one transformer removed in open delta or Vee connection meeting 58 percent of the balanced load. Three phase units cannot have this facility. Mixed single phase loading is not possible due to the absence of neutral.

Star/mesh(Dy or Yd ) This arrangement is very common for power supply transformers. The delta winding permits triplen harmonic currents to circulate in the closed path and attenuates them.

Zig zag/ star (ZY1 or Zy11) Zigzag connection is obtained by inter connection of phases. 4-wire system is possible on both sides. Unbalanced loading is also possible. Oscillating neutral problem is absent in this connection. This connection requires 15% more turns for the same voltage on the zigzag side and hence costs more.

Generally speaking a bank of three single phase transformers cost about 15% more than their 3-phase counter part. Also, they occupy more space. But the spare capacity cost will be less and single phase units are easier to transport.

Mesh connected three phase transformers resemble 3- single phase units but kept in a common tank. In view of this single tank, the space occupied is less. Other than that there is no big difference. The 3-phase core type transformer on the other hand has a simple core arrangement. The three limbs are equal in cross section. Primary and secondary of each
phase are housed on the same limb. The flux setup in any limb will return through the other two limbs as the mmf of those limbs are in the directions so as to aid the same. Even though magnetically this is not a symmetrical arrangement, as the reluctance to the flux setup by side limbs is different from that of the central limb, it does not adversely affect the performance. This is due to the fact that the magnetizing current itself forms a small fraction of the total phase current drawn on load. The added advantage of 3-phase core is that it can tolerate substantially large value of 3rd harmonic mmf without affecting the performance. The 3rd harmonic mmf of the three phases will be in phase and hence rise in all the limbs together.

The 3rd harmonic flux must therefore find its path through the air. Due to the high reluctance of the air path even a substantially large value of third harmonic mmf produces negligible value of third harmonic flux. Similarly unbalanced operation of the transformer with large zero sequence fundamental mmf content also does not affect its performance. Even with Yy type of poly phase connection without neutral connection the oscillating neutral does not occur with these cores. Finally, three phase cores themselves cost less than three single phase units due to compactness.

Parallel operation of one phase and two phase transformers

By parallel operation we mean two or more transformers are connected to the same supply bus bars on the primary side and to a common bus bar/load on the secondary side.

Such requirement is frequently encountered in practice. The reasons that necessitate parallel operation are as follows.

1. Non-availability of a single large transformer to meet the total load requirement.

2. The power demand might have increased over a time necessitating augmentation of the capacity. More transformers connected in parallel will then be pressed into service.

3. To ensure improved reliability. Even if one of the transformers gets into a fault or is taken out for maintenance/repair the load can continued to be serviced.
4. To reduce the spare capacity. If many smaller size transformers are used one machine can be used as spare. If only one large machine is feeding the load, a spare of similar rating has to be available. The problem of spares becomes more acute with fewer machines in service at a location.

5. When transportation problems limit installation of large transformers at site, it may be easier to transport smaller ones to site and work them in parallel.

Fig. 37 shows the physical arrangement of two single phase transformers working in parallel on the primary side. Transformer A and Transformer B are connected to input voltage bus bars. After ascertaining the polarities they are connected to output/load bus bars. Certain conditions have to be met before two or more transformers are connected in parallel and share a common load satisfactorily. They are,
1. The voltage ratio must be the same.

2. The per unit impedance of each machine on its own base must be the same.

3. The polarity must be the same, so that there is no circulating current between the transformers.

4. The phase sequence must be the same and no phase difference must exist between the voltages of the two transformers.

These conditions are examined first with reference to single phase transformers and then the three phase cases are discussed.

**Same voltage ratio** Generally the turns ratio and voltage ratio are taken to be the same. If the ratio is large there can be considerable error in the voltages even if the turns ratios are the same. When the primaries are connected to same bus bars, if the secondaries do not show the same voltage, paralleling them would result in a circulating current between the secondaries. Reflected circulating current will be there on the primary side also. Thus even without connecting a load considerable current can be drawn by the transformers and they produce copper losses. In two identical transformers with percentage impedance of 5 percent, a no-load voltage difference of one percent will result in a circulating current of 10 percent of full load current. This circulating current gets added to the load current when the load is connected resulting in unequal sharing of the load. In such cases the combined full load of the two transformers can never be met without one transformer getting overloaded.

**Per unit impedance**

Transformers of different ratings may be required to operate in parallel. If they have to share the total load in proportion to their ratings the larger machine has to draw more current. The voltage drop across each machine has to be the same by virtue of their connection at the input and the output ends. Thus the larger machines have smaller impedance and smaller machines must have larger ohmic impedance. Thus the impedances must be in the inverse ratios of the ratings. As the voltage drops must be the same the per
unit impedance of each transformer on its own base, must be equal. In addition if active and reactive power are required to be shared in proportion to the ratings the impedance angles also must be the same. Thus we have the requirement that per unit resistance and per unit reactance of both the transformers must be the same for proper load sharing.

**Polarity of connection**

The polarity of connection in the case of single phase transformers can be either same or opposite. Inside the loop formed by the two secondaries the resulting voltage must be zero. If wrong polarity is chosen the two voltages get added and short circuit results. In the case of polyphase banks it is possible to have permanent phase error between the phases with substantial circulating current. Such transformer banks must not be connected in parallel. The turns ratios in such groups can be adjusted to give very close voltage ratios but phase errors cannot be compensated. Phase error of 0.6 degree gives rise to one percent difference in voltage. Hence poly phase transformers belonging to the same vector group alone must be taken for paralleling.

Transformers having \(-30^\circ\) angle can be paralleled to that having \(+30^\circ\) angle by reversing the phase sequence of both primary and secondary terminals of one of the transformers. This way one can overcome the problem of the phase angle error.

**Phase sequence**

The phase sequence of operation becomes relevant only in the case of poly phase systems. The poly phase banks belonging to same vector group can be connected in parallel. A transformer with \(+30^\circ\) phase angle however can be paralleled with the one with \(-30^\circ\) phase angle, the phase sequence is reversed for one of them both at primary and secondary terminals. If the phase sequences are not the same then the two transformers cannot be connected in parallel even if they belong to same vector group. The phase sequence can be found out by the use of a phase sequence indicator.

Performance of two or more single phase transformers working in parallel can be computed using their equivalent circuit. In the case of poly phase banks also the approach is
identical and the single phase equivalent circuit of the same can be used. Basically two cases arise in these problems. Case A: when the voltage ratio of the two transformers is the same and Case B: when the voltage ratios are not the same. These are discussed now in sequence.

Case A: Equal voltage ratios

Always two transformers of equal voltage ratios are selected for working in parallel. This way one can avoid a circulating current between the transformers. Load can be switched on subsequently to these bus bars. Neglecting the parallel branch of the equivalent circuit the above connection can be shown as in Fig. 38(a),(b). The equivalent circuit is drawn in terms of the secondary parameters. This may be further simplified as shown under Fig. 38(c). The voltage drop across the two transformers must be the same by virtue of common connection at input as well as output ends. By inspection the voltage equation for the drop can be
Figure 38: Equivalent Circuit for Transformers working in Parallel - Simplified circuit and Further simplification for identical voltage ratio
If the terminal voltage is $V = IZL$ then the active and reactive power supplied by each of the two transformers is given by

From the above it is seen that the transformer with higher impedance supplies lesser load current and vice versa. If transformers of dissimilar ratings are paralleled the transformer with larger rating shall have smaller impedance as it has to produce the same drop as the other transformer, at a larger current the ohmic values of the impedances must be in the inverse ratio of the ratings of the transformers. $IAZ_A = IBZ_B$, therefore $IA/IB = Z_B/Z_A$.

If the terminal voltage is $V = IZL$ then the active and reactive power supplied by each of the two transformers is given by

$$\begin{align*}
    P_A &= \text{Real}(VI_A^*) \quad \text{and} \quad Q_A = \text{Imag}(VI_A^*) \quad \text{and} \\
    P_B &= \text{Real}(VI_B^*) \quad \text{and} \quad Q_B = \text{Imag}(VI_B^*)
\end{align*}$$

From the above it is seen that the transformer with higher impedance supplies lesser load current and vice versa. If transformers of dissimilar ratings are paralleled the transformer with larger rating shall have smaller impedance as it has to produce the same drop as the other transformer, at a larger current the ohmic values of the impedances must be in the inverse ratio of the ratings of the transformers. $IAZ_A = IBZ_B$, therefore $IA/IB = Z_B/Z_A$. 
Expressing the voltage drops in p.u basis, we aim at the same per unit drops at any load for the transformers. The per unit impedances must therefore be the same on their respective bases.

Fig shows the phasor diagram of operation for these conditions. The drops are magnified and shown to improve clarity. It is seen that the total voltage drop inside the transformers is \( V \) but the currents \( I_A \) and \( I_B \) are forced to have a different phase angle due to the difference in the internal power factor angles \( \phi_A \) and \( \phi_B \). This forces the active and reactive components of the currents drawn by each transformer to be different (even in the case when current in each transformer is the same). If we want them to share the load current in proportion to their ratings, their percentage (or p.u) impedances must be the same. In order to avoid any divergence and to share active and reactive powers also properly, \( \Theta_A = \Theta_B \). Thus the condition for satisfactory parallel operation is that the p.u resistances and p.u reactance must be the same on their respective bases for the two
transformers. To determine the sharing of currents and power either p.u parameters or ohmic values can be used.

**Case B : Unequal voltage ratios**

![Figure 40: Equivalent Circuit for unequal Voltage Ratio](image)

One may not be able to get two transformers of identical voltage ratio inspite of ones best efforts. Due to manufacturing differences, even in transformers built as per the same design, the voltage ratios may not be the same. In such cases the circuit representation for parallel operation will be different as shown in Fig. 40. In this case the two input voltages cannot be merged to one, as they are different. The load brings about a common connection at the output side. EA and EB are the no-load secondary emf. ZL is the load impedance at the secondary terminals. By inspection the voltage equation can be written as below:
\[ E_A = I_A Z_A + (I_A + I_B) Z_L = V + I_A Z_A \cdot \]
\[ E_B = I_B Z_B + (I_A + I_B) Z_L = V + I_B Z_B \cdot \]

Solving the two equations the expression for \( I_A \) and \( I_B \) can be obtained as

\[
I_A = \frac{E_A Z_B + (E_A - E_B) Z_L}{Z_A Z_B + Z_L (Z_A + Z_B)} \quad \text{and} \\
I_B = \frac{E_B Z_A + (E_B - E_A) Z_L}{Z_A Z_B + Z_L (Z_A + Z_B)}
\]

\( Z_A \) and \( Z_B \) are phasors and hence there can be angular difference also in addition to the difference in magnitude. When load is not connected there will be a circulating current between the transformers. The currents in that case can be obtained by putting \( Z_L = 1 \) (after dividing the numerator and the denominator by \( Z_L \)). Then,

\[ I_A = -I_B = \frac{(E_A - E_B)}{(Z_A + Z_B)} \]

If the load impedance becomes zero as in the case of a short circuit, we have,

\[
I_A = \frac{E_A}{Z_A} \quad \text{and} \quad I_B = \frac{E_B}{Z_B}
\]

Instead of the value of \( Z_L \) if the value of \( V \) is known, the currents can be easily determined

\[
I_A = \frac{E_A - V}{Z_A} \quad \text{and} \quad I_B = \frac{E_B - V}{Z_B}
\]
If more than two transformers are connected across a load then the calculation of load currents following the method suggested above involves considerable amount of computational labor. A simpler and more elegant method for the case depicted in Fig. 41 is given below. It is known by the name parallel generator theorem.

\[
I_L = I_A + I_B + I_C + \ldots
\]

But:

\[
I_A = \frac{E_A - V}{Z_A}, \quad I_B = \frac{E_B - V}{Z_B}, \quad I_C = \frac{E_C - V}{Z_C}
\]

\[
V = I_L Z_L
\]

Combining these equations

\[
\frac{V}{Z_L} = \frac{E_A - V}{Z_A} + \frac{E_B - V}{Z_B} + \frac{E_C - V}{Z_C} + \ldots
\]
Grouping the terms together

\[
V \left( \frac{1}{Z_L} + \frac{1}{Z_A} + \frac{1}{Z_B} + \frac{1}{Z_C} + \ldots \right) = \frac{E_A}{Z_A} + \frac{E_B}{Z_B} + \frac{E_C}{Z_C} + \ldots \\
= I_{SCA} + I_{SCB} + I_{SCC} + \ldots \\
\left( \frac{1}{Z_L} + \frac{1}{Z_A} + \frac{1}{Z_B} + \frac{1}{Z_C} + \ldots \right) = \frac{1}{Z} \\
V = Z(I_{SCA} + I_{SCB} + I_{SCC} + \ldots)
\]

From this \( V \) can be obtained. Substituting \( V \) in Eqn. 98, \( I_A \), \( I_B \) etc can be obtained. Knowing the individual current phasor, the load shared by each transformer can be computed.
UNIT 5

**Basic Concepts Of Three Phase Induction Machines:** Concept of rotating magnetic field. Principle of operation, construction, classification and types - single-phase, three-phase, squirrel-cage, slip-ring. Slip, torque, torque-slip characteristic covering motoring, generating and braking regions of operation. Maximum torque.  

7 Hours

**Three-phase Induction Motor**

A three-phase balanced winding in the stator of the Induction motor (IM) is shown in Fig. 29.1 (schematic form). In a three-phase balanced winding, the number of turns in three windings, is equal, with the angle between the adjacent phases, say R & Y, is 120 degree (electrical). Same angle of 120 (elec.) is also between the phases, Y & B.

A three-phase balanced voltage, with the phase sequence as R-Y-B, is applied to the above winding. In a balanced voltage, the magnitude of the voltage in each phase, assumed to be in star in this case, is equal, with the phase angle of the voltage between the adjacent phases, say R & Y, being 120. Schematic diagram of 3 phase Induction motor Rotating Magnetic Field

![Schematic diagram of 3 phase Induction motor Rotating Magnetic Field](image-url)
The three phases of the stator winding (balanced) carry balanced alternating (sinusoidal) currents as shown in Fig.

\[ i_R = I_m \cos \omega t \]
\[ i_Y = I_m \cos (\omega t - 120^\circ) \]
\[ i_B = I_m \cos (\omega t + 120^\circ) = I_m \cos (\omega t - 240^\circ) \]

Three pulsating mmf waves are now set up in the air-gap, which have a time phase difference of from each other. These mmf’s are oriented in space along the magnetic axes of the phases, R, Y & B, as illustrated by the concentrated coils in Fig. Please note that 120° pole stator is shown, with the angle between the adjacent phases, R & Y as , in both mechanical and electrical terms. Since the magnetic axes are located apart in space from each other, the three mmf’s are expresses mathematically

\[ F_R = F_m \cos \omega t \cos \theta \]
\[ F_Y = F_m \cos (\omega t - 120^\circ) \cos (\theta - 120^\circ) \]
\[ F_B = F_m \cos (\omega t + 120^\circ) \cos (\theta + 120^\circ) \]
wherein it has been considered that the three mmf waves differ progressively in time phase by, i.e. \( {120}^\circ/2\pi \) rad (elect.), and are separated in space phase by, i.e. \( {120}^\circ/2\pi \) rad (elect.). The resultant mmf wave, which is the sum of three pulsating mmf waves, is

\[
F = F_R + F_Y + F_B
\]

Substituting the values,

\[
F(\theta, t) = F_m [\cos \omega t \cos \theta + \cos (\omega t - 120^\circ) \cos (\theta - 120^\circ) + \cos (\omega t + 120^\circ) \cos (\theta + 120^\circ)]
\]

The first term of this expression is \( \cos \omega t \cos \theta = 0.5 [\cos (\theta - \omega t) + \cos (\theta + \omega t)] \)

The second term is

\[
\cos (\omega t - 120^\circ) \cos (\theta - 120^\circ) = 0.5 [\cos (\theta - \omega t) + \cos (\theta + \omega t - 240^\circ)]
\]

Similarly, the third term can be rewritten in the form shown.

The expression is

\[
F(\theta, t) = 1.5 F_m \cos(\theta - \omega t)
\]

\[+ 0.5 F_m [\cos (\theta + \omega t) + \cos (\theta + \omega t - 240^\circ) + \cos (\theta + \omega t + 240^\circ)]\]

Note that

\[
\cos (\theta + \omega t - 240^\circ) = \cos (\theta + \omega t + 120^\circ), \text{ and}
\]

\[
\cos (\theta + \omega t + 240^\circ) = \cos (\theta + \omega t - 120^\circ).
\]

If these two terms are added, then

\[\cos (\theta + \omega t + 120^\circ) + \cos (\theta + \omega t - 120^\circ) = -\cos (\theta + \omega t)\]

So, in the earlier expression, the second part of RHS within the capital bracket is zero. In other words, this part represents three unit phasors with a progressive phase difference of 120°, and therefore add up to zero. Hence, the resultant mmf is

\[
F(\theta, t) = 1.5 F_m \cos(\theta - \omega t)
\]

So, the resultant mmf is distributed in both space and time. It can be termed as a rotating magnetic field with sinusoidal space distribution, whose space phase angle changes linearly with time as \( \omega t \). It therefore rotates at a constant angular speed of \( \omega \) rad (elect.)/s. This angular speed is called synchronous angular speed (\( \omega_s \)).

The peak value of the resultant mmf is \( F_{peak} = 1.5 F_m \). The value of \( F_m \) depends on No. of turns/phase, winding current, No. of poles, and winding factor. At \( \omega t = 0 \), i.e. when the current in R phase has maximum positive value, \( F(\theta, 0) = 1.5 F_m \cos \theta \), i.e. the mmf wave has its peak value (at \( \theta = 0 \)) lying on the axis of R phase, when it carries maximum positive current. At \( \omega t = 2\pi/3 \) (120°), the phase Y (assumed lagging) has its positive current maximum, so that the peak of the rotating magnetic field (mmf) lying on the axis of Y phase. By the same argument, the peak of the mmf coincides with the axis of phase B at \( \omega t = 4\pi/3 \) (240°). It is, therefore, seen that the resultant mmf moves from the axis of the leading phase to that of the lagging phase, i.e. from phase R towards phase Y, and then phase B, when the phase sequence of the currents is R-Y-B (R leads Y, and Y leads B). As described in brief later, the direction of rotation of the resultant mmf is reversed by simply changing the phase sequence of currents.
Alternatively, this production of rotating magnetic field can be shown by the procedure described. As stated earlier, the input voltage to three-phase balanced winding of the stator is a balanced one with the phase sequence (R-Y-B). This is shown in the sinusoidal voltage waveforms of the three phases, R, Y & B.

A two-pole, three-phase balanced winding in the stator of IM is shown in Fig. 29.4(i)- (a-d), where the winding of each phase, say for example, \((RR′)\) is assumed to be concentrated in one slot each, both for forward and return conductors, with required no. as needed. Same is the case for other two phases. Please note that the angle of is same in both mechanical (as shown) and also electrical terms, as no. of poles is \(120°\) only. The two (forward and return) parts of the winding in each phase, say R are referred as \(R\) and \(R′\) respectively. So, also for two other phases, Y & B as shown.
Induction machine – the rotor voltage that produces the rotor current and the rotor magnetic field is induced in the rotor windings rather than being physically connected by wires. No dc field current is required to run the machine.

1. Induction Motor Construction

There are basically 2 types of rotor construction:

a) Squirrel Cage - no windings and no slip rings

b) Wound rotor - It has 3 phase windings, usually Y connected, and the winding ends are connected via slip rings.

Wound rotor are known to be more expensive due to its maintenance cost to upkeep the slip rings, carbon brushes and also rotor windings.

Cutaway diagram of a typical large cage rotor induction motor is shown below.
Sketch of cage rotor

Typical wound rotor for induction motors.
Cutaway diagram of a wound rotor induction motor.

Construction of Three-phase Induction Motor

Schematic Diagram of Stator Windings
This is a rotating machine, unlike the transformer, described in the previous module, which is a static machine. Both the machines operate on ac supply. This machine mainly works as a motor, but it can also be run as a generator, which is not much used. Like all rotating machines, it consists of two parts — stator and rotor. In the stator (Fig. 30.1), the winding used is a balanced three-phase one, which means that the number of turns in each phase, connected in star/delta, is equal. The windings of the three phases are placed (electrical) apart, the mechanical angle between the adjacent phases being $\frac{\pi}{p}$, where $p$ is no. of poles. For a 4-pole ($p = 4$) stator, the mechanical angle between the winding of the adjacent phases, is $\alpha = \frac{\pi}{4} = \frac{60}{120} \times 4 = 90^\circ$, as shown in Fig. The conductors, mostly multi-turn, are placed in the slots, which may be closed, or semi-closed, to keep the leakage inductance low. The start and return parts of the winding are placed nearly , or $\beta = 180^\circ - 90^\circ = 90^\circ$ apart. The angle of short chording ($\beta$) is nearly equal to , or close to that value. The short chording results in reducing the amount of copper used for the winding, as the length of the conductor needed for overhang part is reduced. There are also other advantages. The section of the stampings used for both stator and rotor, is shown in Fig. The core is needed below the teeth to reduce the reluctance of the magnetic path, which carries the flux in the motor (machine). The stator is kept normally inside a support.

There are two types of rotor used in IM, viz. squirrel cage and wound (slip-ring) one. The cage rotor (Fig. 30.3a) is mainly used, as it is cheap, rugged and needs little or no maintainance. It consists of copper bars placed in the slots of the rotor, short circuited at the two ends by end rings, brazed with the bars. This type of rotor is equivalent to a wound (slip-ring) one, with the advantage that this may be used for the stator with different no. of poles. The currents in the bars of a cage rotor, inserted inside the stator, follow the pattern of currents in the stator winding, when the motor (IM) develops torque, such that no. of poles in the rotor is same as that in the stator. If the stator winding of IM is changed, with no. of poles for the new one being different from the earlier one, the cage rotor used need not be changed, thus, can be same, as the current pattern in the rotor bars changes. But the no. of poles in the rotor due to the above currents in the bars is same as no. of poles in the new stator winding. The only problem here is that the equivalent resistance of the rotor is
constant. So, at the design stage, the value is so chosen, so as to obtain a certain value of the starting torque, and also the slip at full load torque is kept within limits as needed.

The other type of rotor, i.e., a wound rotor (slip ring) used has a balanced three-phase winding (Fig. 30.3b), being same as the stator winding, but no. of turns used depends on the voltage in the rotor. The three ends of the winding are brought at the three slip-rings, at which points external resistance can be inserted to increase the starting torque requirement. Other three ends are shorted inside. The motor with additional starting resistance is costlier, as this type of rotor is itself costlier than the cage rotor of same power rating, and additional cost of the starting resistance is incurred to increase the starting torque as required. But the slip at full load torque is lower than that of a cage rotor with identical rating, when no additional resistance is used, with direct short-circuiting at the three slip-ring terminals. In both types of rotor, below the teeth, in which bars of a cage rotor, or the conductors of the rotor winding, are placed, lies the iron core, which carries the flux as is the case of the core in the stator. The shaft of the rotor passes below the rotor core. For large diameter of the rotor, a spider is used between the rotor core and the shaft. For a wound (slip-ring) rotor, the rotor winding must be designed for same no. of poles as used for the stator winding. If the no. of poles in the rotor winding is different from no. of poles in the stator winding, no torque will be developed in the motor. It may be noted that this was not the case with cage rotor, as explained earlier.

The wound rotor (slip ring) shown in Fig b is shown as star-connected, whereas the rotor windings can also be connected in delta, which can be converted into its equivalent star configuration. This shows that the rotor need not always be connected in star as shown. The No. of rotor turns changes, as the delta-connected rotor is converted into star-connected equivalent. This point may be kept in mind, while deriving the equivalent circuit as shown in the next lesson (#31), if the additional resistance (being in star) is connected through the slip rings, in series with the rotor winding.
Squirrel Cage Rotor

Slip Ring Rotor

The wound rotor (slip ring) shown in Fig.(b) is shown as star-connected, whereas the rotor windings can also be connected in delta, which can be converted into its equivalent star configuration. This shows that the rotor need not always be connected in star as shown. The No. of rotor turns changes, as the delta-connected rotor is converted into star-connected equivalent, if the additional resistance (being in star) is connected through the slip rings, in series with the rotor winding.
Principle of Operation

The balanced three-phase winding of the stator is supplied with a balanced three-phase voltage. As shown in the previous lesson (#29), the current in the stator winding produces a rotating magnetic field, the magnitude of which remains constant. The axis of the magnetic field rotates at a synchronous speed \((\text{pfns})/2\)\(^{=}\), a function of the supply frequency \((f)\), and number of poles \((p)\) in the stator winding. The magnetic flux lines in the air gap cut both stator and rotor (being stationary, as the motor speed is zero) conductors at the same speed. The emfs in both stator and rotor conductors are induced at the same frequency, i.e. line or supply frequency, with No. of poles for both stator and rotor windings (assuming wound one) being same. The stator conductors are always stationary, with the frequency in the stator winding being same as line frequency. As the rotor winding is short-circuited at the slip-rings, current flows in the rotor windings. The electromagnetic torque in the motor is in the same direction as that of the rotating magnetic field, due to the interaction between the rotating flux produced in the air gap by the current in the stator winding, and the current in the rotor winding. This is as per Lenz’s law, as the developed torque is in such direction that it will oppose the cause, which results in the current flowing in the rotor winding. This is irrespective of the rotor type used – cage or wound one, with the cage rotor, with the bars short-circuited by two end-rings, is considered equivalent to a wound one. The current in the rotor bars interacts with the air-gap flux to develop the torque, irrespective of the no. of poles for which the winding in the stator is designed. Thus, the cage rotor may be termed as universal one. The induced emf and the current in the rotor are due to the relative velocity between the rotor conductors and the rotating flux in the air-gap, which is maximum, when the rotor is stationary (). As the rotor starts rotating in the same direction, as that of the rotating magnetic field due to production of the torque as stated earlier, the relative velocity decreases, along with lower values of induced emf and current in the rotor. If the rotor speed is equal that of the rotating magnetic field, which is termed as synchronous speed, and also in the same direction, the relative velocity is \(0.0=\text{rnzero}\), which causes both the induced emf and current in the rotor to be reduced to zero. Under this condition, torque will not be produced. So, for production of positive
(motoring) torque, the rotor speed must always be lower than the synchronous speed. The rotor speed is never equal to the synchronous speed in an IM. The rotor speed is determined by the mechanical load on the shaft and the total rotor losses, mainly comprising of copper loss. The difference between the synchronous speed and rotor speed, expressed as a ratio of the synchronous speed, is termed as ‘slip’ in an IM. So, slip ($s$) in pu is

$$s = \frac{n_s - n_r}{n_s} = 1 - \frac{n_r}{n_s}$$

or, $n_r = (1 - s) \cdot n_s$

where, $n_s$ and $n_r$ are synchronous and rotor speeds in rev/s.

In terms of $N_s = 60 \times n_s$ and $N_r = 60 \times n_r$, both in rev/min (rpm), slip is

$$s = \frac{N_s - N_r}{N_s}$$

If the slip is expressed in %, then $s = \frac{(N_s - N_r)}{N_s} \times 100$

Normally, for torques varying from no-load ($\approx$ zero) to full load value, the slip is proportional to torque. The slip at full load is $4\text{-}5\%$ (0.04-0.05).
An alternative explanation for the production of torque in a three-phase induction motor is given here, using two rules (right hand and left hand) of Fleming. The stator and rotor, along with air-gap, is shown in Fig. 30.4a. Both stator and rotor is shown there as surfaces, but without the slots as given in Fig, 30.2. Also shown is the path of the flux in the air gap. This is for a section, which is under North pole, as the flux lines move from stator to rotor. The rotor conductor shown in the figure is at rest, i.e., zero speed (stand-still). The rotating magnetic field moves past the conductor at synchronous speed in the clockwise direction. Thus, there is relative movement between the flux and the rotor conductor. Now, if the magnetic field, which is rotating, is assumed to be at standstill as shown in Fig. 30.4b, the conductor will move in the direction shown. So, an emf is induced in the rotor conductor as per Faraday’s law, due to change in flux linkage. The direction of the induced emf as shown in the figure can be determined using Fleming’s right hand rule.

As described earlier, the rotor bars in the cage rotor are short circuited via end rings. Similarly, in the wound rotor, the rotor windings are normally short-circuited externally via the slip rings. In both cases, as emf is induced in the rotor conductor (bar), current flows there, as it is short circuited. The flux in the air gap, due to the current in the rotor conductor is shown in Fig. 30.4c. The flux pattern in the air gap, due to the magnetic fields produced by the stator windings and the current carrying rotor conductor, is shown in Fig.
304d. The flux lines bend as shown there. The property of the flux lines is to travel via shortest path as shown in Fig. 30.4a. If the flux lines try to move to form straight line, then the rotor conductor has to move in the direction of the rotating magnetic field, but not at the same speed, as explained earlier. The current carrying rotor conductor and the direction of flux are shown in Fig. 30.4e. It is known that force is produced on the conductor carrying current, when it is placed in a magnetic field. The direction of the force on the rotor conductor is obtained by using Fleming’s left hand rule, being same as that of the rotating magnetic field. Thus, the rotor experiences a motoring torque in the same direction as that of the rotating magnetic field. This briefly describes how torque is produced in a three-phase induction motor.

The frequency of the induced emf and current in the rotor As given earlier, both the induced emf and the current in the rotor are due to the relative velocity between the rotor conductors and the rotating flux in the air-gap, the speed of which is the synchronous speed. Above frequency is small. Taking an example, with full load slip as 4% (0.04), and supply (line) frequency as 50 Hz, the frequency (Hz) of the rotor induced emf and current, is 0.20.5004.0=×rf, which is very small, whereas the frequency (f) of the stator induced emf and current is 50 Hz, i.e. line frequency. At standstill, i.e. rotor stationary (0.0=rm), the rotor frequency is same as line frequency, as shown earlier, with slip [s = 1.0 (100%)]. The reader is requested to read the next lesson (#31), where some additional points are included in this matter.
8.10 Rotor Torque

The torque $T$ developed by the rotor is directly proportional to:

(i) rotor current
(ii) rotor e.m.f.
(iii) power factor of the rotor circuit

\[ \therefore T \propto E_2 I_2 \cos \phi_2 \]

or

\[ T = K E_2 I_2 \cos \phi_2 \]

where

- $I_2 = \text{rotor current at standstill}$
- $E_2 = \text{rotor e.m.f. at standstill}$
- $\cos \phi_2 = \text{rotor p.f. at standstill}$

**Note.** The values of rotor e.m.f., rotor current and rotor power factor are taken for the given conditions.

8.11 Starting Torque ($T_s$)

Let

- $E_2 = \text{rotor e.m.f. per phase at standstill}$
- $X_2 = \text{rotor reactance per phase at standstill}$
- $R_2 = \text{rotor resistance per phase}$

Rotor impedance/phase,

\[ Z_2 = \sqrt{R_2^2 + X_2^2} \quad \text{...at standstill} \]

Rotor current/phase,

\[ I_2 = \frac{E_2}{Z_2} = \frac{E_2}{\sqrt{R_2^2 + X_2^2}} \quad \text{...at standstill} \]

Rotor p.f.,

\[ \cos \phi_2 = \frac{R_2}{Z_2} = \frac{R_2}{\sqrt{R_2^2 + X_2^2}} \quad \text{...at standstill} \]

\[ \therefore \text{Starting torque, } T_s = K E_2 I_2 \cos \phi_2 \]

\[ = K E_2 \times \frac{E_2}{\sqrt{R_2^2 + X_2^2}} \times \frac{R_2}{\sqrt{R_2^2 + X_2^2}} \]

\[ = \frac{K E_2^2 R_2}{R_2^2 + X_2^2} \]
Generally, the stator supply voltage $V$ is constant so that flux per pole $f$ set up by the stator is also fixed. This in turn means that e.m.f. $E_2$ induced in the rotor will be constant.

$$\therefore \quad T_s = \frac{K_1 R_2}{R_2^2 + X_2^2} = \frac{K_1 R_2}{Z_2^2}$$

It is clear that the magnitude of starting torque would depend upon the relative values of $R_2$ and $X_2$ i.e., rotor resistance/phase and standstill rotor reactance/phase.

It can be shown that $K = \frac{3}{2\pi} N_s$.

$$\therefore \quad T_s = \frac{3}{2\pi N_s} \cdot \frac{E_2^2 R_2}{R_2^2 + X_2^2}$$

Note that here $N_s$ is in r.p.s.

**Condition for Maximum Starting Torque**

It can be proved that starting torque will be maximum when rotor resistance/phase is equal to standstill rotor reactance/phase.

Now

$$T_s = \frac{K_1 R_2}{R_2^2 + X_2^2} \quad \text{(i)}$$

Differentiating eq. (i) w.r.t. $R_2$ and equating the result to zero, we get,

$$\frac{dT_s}{dR_2} = K_1 \left[ \frac{1}{R_2^2 + X_2^2} - \frac{R_2 (2 R_2)}{(R_2^2 + X_2^2)^2} \right] = 0$$

or

$$R_2^2 + X_2^2 = 2R_2^2$$

or

$$R_2 = X_2$$

Hence starting torque will be maximum when: Rotor resistance/phase = Standstill rotor reactance/phase Under the condition of maximum starting torque, $f_2 = 45^\circ$ and rotor power
factor is 0.707 lagging [See Fig. (ii)]. Fig. (i) shows the variation of starting torque with rotor resistance. As the rotor resistance is increased from a relatively low value, the starting torque increases until it becomes maximum when \( R_2 = X_2 \). If the rotor resistance is increased beyond this optimum value, the starting torque will decrease.

![Starting Torque Curve](image)

### Starting Torque of 3-Phase Induction Motors

The rotor circuit of an induction motor has low resistance and high inductance. At starting, the rotor frequency is equal to the stator frequency (i.e., 50 Hz) so that rotor reactance is large compared with rotor resistance. Therefore, rotor current lags the rotor e.m.f. by a large angle, the power factor is low and consequently the starting torque is small. When resistance is added to the rotor circuit, the rotor power factor is improved which results in improved starting torque. This, of course, increases the rotor impedance and, therefore, decreases the value of rotor current but the effect of improved power factor predominates and the starting torque is increased.

(i) **Squirrel-cage motors.** Since the rotor bars are permanently short circuited, it is not possible to add any external resistance in the rotor circuit at starting. Consequently, the
stalling torque of such motors is low. Squirrel 196 cage motors have starting torque of 1.5 to 2 times the full-load value with starting current of 5 to 9 times the full-load current.

(ii) **Wound rotor motors.** The resistance of the rotor circuit of such motors can be increased through the addition of external resistance. By inserting the proper value of external resistance (so that \( R_2 = X_2 \)), maximum starting torque can be obtained. As the motor accelerates, the external resistance is gradually cut out until the rotor circuit is short-circuited on itself for running conditions.

**Maximum Torque under Running Conditions**

\[
T_r = \frac{K_2 s R_2}{R_2^2 + s^2 X_2^2} \quad (i)
\]

In order to find the value of rotor resistance that gives maximum torque under running conditions, differentiate exp. (i) w.r.t. \( s \) and equate the result to zero i.e.,

\[
\frac{dT_r}{ds} = K_2 \left[ R_2 \left( R_2^2 + s^2 X_2^2 \right) - 2s X_2 \left( s R_2 \right) \right] = 0
\]

or

\[
\left( R_2^2 + s^2 X_2^2 \right) - 2s X_2^2 = 0
\]

or

\[
R_2^2 = s^2 X_2^2
\]

or

\[
R_2 = s X_2
\]

Thus for maximum torque \((T_m)\) under running conditions:

Rotor resistance/phase = Fractional slip \( \times \) Standstill rotor reactance/phase

Now

\[
T_r \propto \frac{s R_2}{R_2^2 + s^2 X_2^2}
\]

… from exp. (i) above

For maximum torque, \( R_2 = s X_2 \). Putting \( R_2 = s X_2 \) in the above expression, the maximum torque \( T_m \) is given by:

\[
T_m \propto \frac{1}{2 X_2}
\]

Slip corresponding to maximum torque, \( s = R_2/X_2 \).
It is evident from the above equations that:

(i) The value of rotor resistance does not alter the value of the maximum torque but only the value of the slip at which it occurs.

(ii) The maximum torque varies inversely as the standstill reactance. Therefore, it should be kept as small as possible.

(iii) The maximum torque varies directly with the square of the applied voltage.

(iv) To obtain maximum torque at starting \((s = 1)\), the rotor resistance must be made equal to rotor reactance at standstill.

**Torque-Slip Characteristics**

The motor torque under running conditions is given by;

\[
T = \frac{K_2 s R_2}{R_2^2 + s^2 X_2^2}
\]

If a curve is drawn between the torque and slip for a particular value of rotor resistance \(R_2\), the graph thus obtained is called torque-slip characteristic. Fig shows a family of torque-slip characteristics for a slip-range from \(s = 0\) to \(s = 1\) for various values of rotor resistance.
The following points may be noted carefully:

(i) At \( s = 0 \), \( T = 0 \) so that torque-slip curve starts from the origin.

(ii) At normal speed, slip is small so that \( s \times X_2 \) is negligible as compared to \( R_2 \).

\[ \therefore \quad T \propto \frac{s}{R_2} \]

\[ \propto s \text{ ... as } R_2 \text{ is constant} \]

Hence torque-slip curve is a straight line from zero slip to a slip that corresponds to full-load.

(iii) As slip increases beyond full-load slip, the torque increases and becomes maximum at \( s = \frac{R_2}{X_2} \). This maximum torque in an induction motor is called pull-out torque or breakdown torque. Its value is at least twice the full-load value when the motor is operated at rated voltage and frequency.

(iv) To maximum torque, the term \( s^2X_2^2 \) increases very rapidly so that \( R_2^2 \) may be neglected as compared to \( s^2X_2^2 \).

\[ \therefore \quad T \propto \frac{s}{s^2X_2^2} \]

\[ \propto \frac{1}{s} \text{ ... as } X_2 \text{ is constant} \]

Thus the torque is now inversely proportional to slip. Hence torque-slip curve is a rectangular hyperbola.

(v) The maximum torque remains the same and is independent of the value of rotor resistance. Therefore, the addition of resistance to the rotor circuit does not change the value of maximum torque but it only changes the value of slip at which maximum torque occurs.
Figure: Coils in the stator

Figure: A wound rotor with slip rings

Figure: slip rings
Figure: squirrel cage rotor
UNIT- 6


Equivalent Circuit

It is often required to make quantitative predictions about the behavior of the induction machine, under various operating conditions. For this purpose, it is convenient to represent the machine as an equivalent circuit under sinusoidal steady state operating conditions. Since the operation is balanced, a single-phase equivalent circuit is sufficient for most purposes. In order to derive the equivalent circuit, let us consider a machine with an open circuited rotor. Since no current can ow and as a consequence no torque can be produced, the situation is like a transformer open-circuited on the secondary (rotor). The equivalent circuit under this condition can be drawn as shown in fig.

Approximate Equivalent Circuit of Induction Motor

As in case of a transformer, the approximate equivalent circuit of an induction motor is obtained by shifting the shunt branch \((R_c - X_m)\) to the input terminals as shown in Fig. This step has been taken on the assumption that voltage drop in \(R_1\) and \(X_1\) is small and the terminal voltage \(V_1\) does not appreciably differ from the induced voltage \(E_1\). Fig shows the approximate equivalent circuit per phase of an induction motor where all values have been referred to primary (i.e., stator).
The above approximate circuit of induction motor is not so readily justified as with the transformer. This is due to the following reasons:

(i) Unlike that of a power transformer, the magnetic circuit of the induction motor has an air-gap. Therefore, the exciting current of induction motor (30 to 40% of full-load current) is much higher than that of the power transformer. Consequently, the exact equivalent circuit must be used for accurate results.

(ii) The relative values of $X_1$ and $X_2$ in an induction motor are larger than the corresponding ones to be found in the transformer. This fact does not justify the use of approximate equivalent circuit.

(iii) In a transformer, the windings are concentrated whereas in an induction motor, the windings are distributed. This affects the transformation ratio.

In spite of the above drawbacks of approximate equivalent circuit, it yields results that are satisfactory for large motors. However, approximate equivalent circuit is not justified for small motors.

**The no-load test**

The behavior of the machine may be judged from the equivalent circuit of fig. The current drawn by the machine causes a stator-impedance drop and the balance voltage is applied across the magnetizing branch. However, since the magnetizing branch impedance is large, the current drawn is small and hence the stator impedance drop is small compared to the applied voltage (rated value). This drop and the power dissipated in the stator resistance are therefore neglected and the total power drawn is assumed to be consumed entirely as core loss. This can also be seen from the approximate equivalent circuit, the use of which is justified by the foregoing arguments. This test therefore enables us to compute the resistance and inductance of the magnetizing branch in the following manner.

Let applied voltage $= V_s$. Then current drawn is given by

$$I_s = \frac{V_s}{R_m} + \frac{V_s}{jX_m}$$
The power drawn is given by

\[
P_s = \frac{V_s^2}{R_m} \Rightarrow R_m = \frac{V_s^2}{P_s}
\]

Vs; Is and Ps are measured with appropriate meters. With Rm known from eqn, Xm can be found from eqn. The current drawn is at low power factor and hence a suitable wattmeter should be used.

**Blocked-rotor Test**

In this test the rotor is prevented from rotation by mechanical means and hence the name. Since there is no rotation, slip of operation is unity, s = 1. The equivalent circuit valid under these conditions is shown in fig. 21(b). Since the current drawn is decided by the resistance and leakage impedances alone, the magnitude can be very high when rated voltage is applied. Therefore in this test, only small voltages are applied just enough to cause rated current to flow. While the current magnitude depends on the resistance and the reactance, the power drawn depends on the resistance. The parameters may then be determined as follows. The source current and power drawn may be written as

\[
I_s = \frac{V_s}{(R_s + R'_r) + j(X_s + X'_r)}
\]

\[
P_s = |I_s|^2(R_s + R'_r)
\]

In the test Vs; Is and Ps are measured with appropriate meters. Above equation enables us to compute(Rs + R0r). Once this is known, (Xs + X0r) may be computed from the equation. Note that this test only enables us to determine the series combination of the resistance and the reactance only and not the individual values. Generally, the individual values are assumed to be equal; the assumption Rs = R'r, and Xs = X'r suffices for most purposes. In practice, there are differences. If more accurate estimates are required IEEE guidelines may be followed which depend on the size of the machine.
Note that these two tests determine the equivalent circuit parameters in a `Stator-referred' sense, i.e., the rotor resistance and leakage inductance are not the actual values but what they 'appear to be' when looked at from the stator. This is sufficient for most purposes as interconnections to the external world are generally done at the stator terminals.
**UNIT 7**


**Double Squirrel-Cage Motors**

One of the advantages of the slip-ring motor is that resistance may be inserted in the rotor circuit to obtain high starting torque (at low starting current) and then cut out to obtain optimum running conditions. However, such a procedure cannot be adopted for a squirrel cage motor because its cage is permanently short-circuited. In order to provide high starting torque at low starting current, double-cage construction is used.

**Construction**

As the name suggests, the rotor of this motor has two squirrel-cage windings located one above the other as shown in Fig.(i).

(i) **The outer winding** consists of bars of smaller cross-section short-circuited by end rings. Therefore, the resistance of this winding is high. Since the outer winding has relatively open slots and a poorer flux path around its bars [See Fig.(ii)], it has a low inductance. Thus the resistance of the outer squirrel-cage winding is high and its inductance is low.

(ii) **The inner winding** consists of bars of greater cross-section short-circuited by end rings. Therefore, the resistance of this winding is low. Since the bars of the inner winding are thoroughly buried in iron, it has a high inductance [See Fig. (8.35 (ii))]. Thus the resistance of the inner squirrel cage winding is low and its inductance is high.
Working

When a rotating magnetic field sweeps across the two windings, equal e.m.f.s are induced in each.

(i) At starting, the rotor frequency is the same as that of the line (i.e., 50 Hz), making the reactance of the lower winding much higher than that of the upper winding. Because of the high reactance of the lower winding, nearly all the rotor current flows in the high-resistance outer cage winding. This provides the good starting characteristics of a high-resistance cage winding. Thus the outer winding gives high starting torque at low starting current.

(ii) As the motor accelerates, the rotor frequency decreases, thereby lowering the reactance of the inner winding, allowing it to carry a larger proportion of the total rotor current. At the normal operating speed of the motor, the rotor frequency is so low (2 to 3 Hz) that nearly all the rotor current flows in the low-resistance inner cage winding. This results in good operating efficiency and speed regulation. Fig. (a) shows the operating characteristics of double squirrel-cage motor.

The starting torque of this motor ranges from 200 to 250 percent of full-load torque with a starting current of 4 to 6 times the full-load value. It is classed as a high-torque, low starting current motor.
Equivalent Circuit of Double Squirrel-Cage Motor

Fig. shows a section of the double squirrel cage motor. Here $R_o$ and $R_i$ are the per phase resistances of the outer cage winding and inner cage winding whereas $X_o$ and $X_i$ are the corresponding per phase standstill reactance. For the outer cage, the resistance is made intentionally high, giving a high starting torque. For the inner cage winding, the resistance is low and the leakage reactance is high, giving a low starting torque but high efficiency on load. Note that in a double squirrel cage motor, the outer winding produces the high starting and accelerating torque while the inner winding provides the running torque at good efficiency. Fig. (i) shows the equivalent circuit for one phase of double cage motor referred to stator. The two cage impedances are effectively in parallel. The resistances and reactances of the outer and inner rotors are referred to the stator. The exciting circuit is accounted for as in a single cage motor. If the magnetizing current ($I_0$) is neglected, then the circuit is simplified to that shown in Fig. (ii).
From the equivalent circuit, the performance of the motor can be predicted. Total impedance as referred to stator is

\[ Z_{OL} = R_1 + jX_1 + \frac{1}{\frac{1}{Z'_{i}} + \frac{1}{Z'_{o}}} = R_1 + jX_1 + \frac{Z'_i Z'_o}{Z'_i + Z'_o} \]
UNIT- 8

(a) Starting and speed Control of Three-phase Induction Motors: Need for starter. Direct on line (DOL), Star-Delta and autotransformer starting. Rotor resistance starting. Soft(electronic) starters. Speed control -voltage, frequency, and rotor resistance. 4 Hours

(b) Single-phase Induction Motor: Double revolving field theory and principle of operation. Types of single-phase induction motors: split-phase, capacitor start, shaded pole motors. Applications. 3 Hours

Starting of 3-Phase Induction Motors

The induction motor is fundamentally a transformer in which the stator is the primary and the rotor is short-circuited secondary. At starting, the voltage induced in the induction motor rotor is maximum (Q s = 1). Since the rotor impedance is low, the rotor current is excessively large. This large rotor current is reflected in the stator because of transformer action. This results in high starting current (4 to 10 times the full-load current) in the stator at low power factor and consequently the value of starting torque is low. Because of the short duration, this value of large current does not harm the motor if the motor accelerates normally. However, this large starting current will produce large line-voltage drop. This will adversely affect the operation of other electrical equipment connected to the same lines. Therefore, it is desirable and necessary to reduce the magnitude of stator current at starting and several methods are available for this purpose.

Starters for Poly Phase Induction Motors

If motor is started with full voltage, the starting torque is good but very large currents, of the order of 5-7 times the full-load current flow which causes objectionable voltage drop in the power supply lines and hence undesirable dip in the supply line voltage. Consequently, the operation of other equipment connected to the same supply line is affected considerably.
If the motor is started with reduced voltage, there is no problem of high currents but it produces an objectionable reduction in the starting torque, on account of the fact that motor torque is proportional to the square of the applied voltage.

**Methods of Starting Squirrel Cage I.M**

There are basic four methods of starting the squirrel cage induction motor using

(a) Direct online starters

(b) Stator Resistor (or reactor) Starters

(c) Auto-transformer Starters

(d) Star-Delta Starters

![Diagram of Direct online starters](image-url)
Methods of Starting Slip-Ring (Wound Rotor) I.M.

Though all the above methods, except D.O.L. where the high currents may damage the rotor windings, can also be employed for starting slip-ring motors, but it is usually not done because the advantages of such motors can’t be fully realized. So the method of adding resistance to the rotor circuit is the most common method for rotor wound I.M. starting.

D.O.L. Starters

The above Figure shows a contactor type D.O.L. starter connected to a motor. As soon as the push-button $S_1$ is pressed, the contactor coil is energized closing its contacts $M_1$, $M_2$ and $M_3$. Then, the motor windings get full supply through back-up fuses $e_1$, $e_2$, $e_3$ and bimetallic relays $O_1$, $O_2$ and $O_3$ and the motor starts running. An auxiliary contact $A$ in $C_1$ retains the contactor in closed position after the release of start switch $S_1$. An overload tripping device $e_1$, working in conjunction with bimetallic relays, is placed in series with the contactor coil, so that during sustained overload, this opens and the motor stops automatically. For stopping the motor any time, a stop button is provided in series with the contactor coil.

Primary Resistor Starter and Reactor Starter

This method consists of connecting the motor to the line voltage through a series resistance in each phase. The resistors are short-circuited when the motor accelerates to the desired speed. Sequence of operation of switches shown in Figure below is:

(a) Initially all switches are open.
(b) Switches 1, 2 and 3 are closed simultaneously and motor starts running with full resistances in series.
(c) Switches 4, 5 and 6 are closed when motor speed picks up and current becomes constant. Finally switches 7, 7, and 9 are closed to cut all resistances and motors attains final steady state speed.
**Advantages:**

(a) It provides closed transition starting, resulting in smooth starting without any transition high current.
(b) A higher p.f. than auto-transformer starters.

Sometimes as an alternative to resistor starting, reactor starting is used. This method is mainly used for large motors.

**Auto-Transformer in the First Step Starters**

In this type of starter, (A. T.) it attains the reduced voltage by means of an auto transformer at the start. After a definite time interval (about 15 sec.), and after the motor accelerates, it is transferred from the reduced voltage to 133 the full voltage in the second step. A. T. are generally provided with voltage drops to give 40%, 60%, 75% and 100% line voltage. The starting current and starting torque depends on the tapping selected. In the third step, the change-over switch may be hand operated or automatic through time relay which connects the motor finally to the line by changing over from position A to B.
Fig: Simple Diagram of Auto-transformer Starter
Advantages of A.T. Starters

(i) Greater efficiency.
(ii) Taps on the transformer allow adjustment of starting torque to meet the particular requirement.

Disadvantages of A. T. Starters

(i) It opens the circuit before the motor is connected directly to the line, thus producing transient current and stresses.
(ii) It reduces the p.f. of the circuit.
(iii) The torque remains constant for the second step, resulting in acceleration which is not smooth.

These disadvantages of open transition in A. T. may be overcome by the use of Korndorfer connection, which introduces another step in starting. On the second step, part of the A. T. remains in series with the stator windings. The third step involves the transfer of the full-voltage without open transition.

Star-Delta Starter

It is cheaper as compared to A. T. starter. This method of starting is used for motors designed to operate normally in delta. The six terminals from the three phases of the stator must be available:

\begin{align*}
    a, A &: \text{Terminals of phase A} \\
    b, B &: \text{Terminals of phase B} \\
    c, C &: \text{Terminals of phase C}
\end{align*}

Commercially, the terminals are marked \(A1, A2; B1, B2\) and \(C1, C2\) respectively. The motor is started with TPDT switch in position 1 and subsequently switched to position 2.

Position 1 : Starting-windings connected in \(Y\)
Position 2 : Running-windings get connected in \(D\)
Fig: Star-Delta Starter
Let $V_L = \text{Line Voltage}$

$I_{SP, y} = \text{Per phase motor current during start (in Y-connection)}$

\[ \therefore I_{L, y} = I_{SP, y} = \frac{V_L}{\sqrt{3} Z_1} \]

(5.1)

[Since rotor circuit behaves almost as short circuit on starting as $R_2' \left( \frac{1}{s} - 1 \right) = 0$.]

If stator winding is $\Delta$-connected, then, with direct switching the per-phase motor current at start would be given by

\[ I_{SP, \Delta} = \frac{V_L}{Z_1} \]

(5.2)

Starting line current

\[ I_{L, \Delta} = \sqrt{3} \quad I_{SP, \Delta} = \frac{\sqrt{3} V_L}{Z_1} \]

(5.3)

It is shown from Eq. (5.1) to (5.2) that

\[
\frac{\text{Starting line current with star-delta starter}}{\text{Starting line current with direct switching in delta}} = \frac{I_{SL, y}}{I_{L, \Delta}} = \frac{\frac{1}{\sqrt{3}}}{\frac{\sqrt{3} V_L}{Z_1}} = \frac{1}{3}
\]

and

\[
\frac{\text{Starting torque with star-delta starter}}{\text{Starting torque with direct switching in delta}} = \frac{\left( \frac{V_L}{\sqrt{3}} \right)^2}{V_L^2} = \frac{1}{3}.
\]
Methods of Starting 3-Phase Induction Motors

The method to be employed in starting a given induction motor depends upon the size of the motor and the type of the motor. The common methods used to start induction motors are:

(i) Direct-on-line starting  (ii) Stator resistance starting  (iii) Autotransformer starting  
(iv) Star-delta starting  (v) Rotor resistance starting

Methods (i) to (iv) are applicable to both squirrel-cage and slip ring motors. However, method (v) is applicable only to slip ring motors. In practice, any one of the first four methods is used for starting squirrel cage motors, depending upon the size of the motor. But slip ring motors are invariably started by rotor resistance starting.

Methods of Starting Squirrel-Cage Motors

Except direct-on-line starting, all other methods of starting squirrel-cage motors employ reduced voltage across motor terminals at starting.

(i) Direct-on-line starting

This method of starting in just what the name implies—the motor is started by connecting it directly to 3-phase supply. The impedance of the motor at standstill is relatively low and when it is directly connected to the supply system, the starting current will be high (4 to 10 times the full-load current) and at a low power factor. Consequently, this method of starting is suitable for relatively small (up to 7.5 kW) machines.

Relation between starting and F.L. torques. We know that:

\[ \text{Rotor input} = 2\pi N_s T - kT \]

But \[ \text{Rotor Cu loss} = s \times \text{Rotor input} \]

\[
\therefore \quad 3(I'_2)^2 R_2 = s \times kT \\
\text{or} \quad T \propto \frac{(I'_2)^2}{s}
\]
If $I_{st}$ is the starting current, then starting torque ($T_{st}$) is

$$T \propto I_{st}^2$$

(\because \text{ at starting } s = 1)

If $I_f$ is the full-load current and $s_f$ is the full-load slip, then,

$$T_f \propto \frac{I_f^2}{s_f}$$

$$\therefore \frac{T_{st}}{T_f} = \left(\frac{I_{st}}{I_f}\right)^2 \times s_f$$

When the motor is started direct-on-line, the starting current is the short-circuit (blocked-rotor) current $I_{sc}$.

$$\therefore \frac{T_{st}}{T_f} = \left(\frac{I_{sc}}{I_f}\right)^2 \times s_f$$

Let us illustrate the above relation with a numerical example. Suppose $I_{sc} = 5I_f$ and full-load slip $s_f=0.04$. Then,

$$\frac{T_{st}}{T_f} = \left(\frac{I_{sc}}{I_f}\right)^2 \times s_f = \left(\frac{5I_f}{I_f}\right)^2 \times 0.04 = (5)^2 \times 0.04 = 1$$

Note that starting current is as large as five times the full-load current but starting torque is just equal to the full-load torque. Therefore, starting current is very high and the starting torque is comparatively low. If this large starting current flows for a long time, it may overheat the motor and damage the insulation.

(ii) **Autotransformer starting**

This method also aims at connecting the induction motor to a reduced supply at starting and then connecting it to the full voltage as the motor picks up sufficient speed. Fig. shows the circuit arrangement for autotransformer starting. The tapping on the autotransformer is so set that when it is in the circuit, 65% to 80% of line voltage is applied to the motor.

At the instant of starting, the change-over switch is thrown to “start” position. This puts the autotransformer in the circuit and thus reduced voltage is applied to the circuit. Consequently, starting current is limited to safe value. When the motor attains about 80% of normal speed, the changeover switch is thrown to 220 “run” position. This takes out the
autotransformer from the circuit and puts the motor to full line voltage. Autotransformer
starting has several advantages viz low power loss, low starting current and less radiated
heat. For large machines (over 25 H.P.), this method of starting is often used. This method
can be used for both star and delta connected motors.

(iii) Star-delta starting

The stator winding of the motor is designed for delta operation and is connected in
star during the starting period. When the machine is up to speed, the connections are
changed to delta. The circuit arrangement for star-delta starting is shown in Fig. The six
leads of the stator windings are connected to the changeover switch as shown. At the instant
of starting, the changeover switch is thrown to “Start” position which connects the stator
windings in star. Therefore, each stator phase gets V/3 volts where V is the line voltage.
This reduces the starting current. When the motor picks up speed, the changeover switch is
thrown to “Run” position which connects the stator windings in delta. Now each stator
phase gets full line voltage V.

The disadvantages of this method are:

(a) With star-connection during starting, stator phase voltage is 1/\sqrt{3} times the line
voltage. Consequently, starting torque is \((1/\sqrt{3})^2\) or \(1/3\) times the value it would have
with D-connection. This is rather a large reduction in starting torque.
(b) The reduction in voltage is fixed. This method of starting is used for medium-size machines (upto about 25 H.P.).

\[ I_{st} = \frac{1}{\sqrt{3}} I_{sc} \]

\[ V/\sqrt{3} \quad Z_{sc} \]

Star (Stator)

\[ \sqrt{3} I_{sc} \quad I_{sc} \quad Z_{sc} \]

Delta (Stator)

**Starting of Slip-Ring Motors**

Slip-ring motors are invariably started by rotor resistance starting. In this method, a variable star-connected rheostat is connected in the rotor circuit through slip rings and full voltage is applied to the stator winding as shown in Fig.

(i) At starting, the handle of rheostat is set in the OFF position so that maximum resistance is placed in each phase of the rotor circuit. This reduces the starting current and at the same time starting torque is increased.
(ii) As the motor picks up speed, the handle of rheostat is gradually moved in clockwise direction and cuts out the external resistance in each phase of the rotor circuit. When the motor attains normal speed, the change-over switch is in the ON position and the whole external resistance is cut out from the rotor circuit.

**Slip-Ring Motors Versus Squirrel Cage Motors**

The slip-ring induction motors have the following advantages over the squirrel cage motors:

(i) High starting torque with low starting current.

(ii) Smooth acceleration under heavy loads.

(iii) No abnormal heating during starting.

(iv) Good running characteristics after external rotor resistances are cut out.

(v) Adjustable speed.

The disadvantages of slip-ring motors are:

(i) The initial and maintenance costs are greater than those of squirrel cage motors.

(ii) The speed regulation is poor when run with resistance in the rotor circuit

**Speed control of Induction Machines**

We have seen the speed torque characteristic of the machine. In the stable region of operation in the motoring mode, the curve is rather steep and goes from zero torque at synchronous speed to the stall torque at a value of slip \( s = \bar{s} \). Normally \( \bar{s} \) may be such that stall torque is about three times that of the rated operating torque of the machine, and hence may be about 0.3 or less. This means that in the entire loading range of the machine, the speed change is quite small. The machine speed is quite sti_ with respect to load changes. The entire speed variation is only in the range \( \text{ns} \) to \((1 - \bar{s})\text{ns}\), \( \text{ns} \) being dependent on supply frequency and number of poles.

The foregoing discussion shows that the induction machine, when operating from mains is essentially a constant speed machine. Many industrial drives, typically for fan or pump applications, have typically constant speed requirements and hence the induction
machine is ideally suited for these. However, the induction machine, especially the squirrel cage type, is quite rugged and has a simple construction. Therefore it is good candidate for variable speed applications if it can be achieve

1. **Speed control by changing applied voltage**

From the torque equation of the induction machine given in eqn. 16, we can see that the torque depends on the square of the applied voltage. The variation of speed torque curves with respect to the applied voltage is shown in fig. These curves show that the slip at maximum torque \( s \) remains same, while the value of stall torque comes down with decrease in applied voltage. The speed range for stable operation remains the same. Further, we also note that the starting torque is also lower at lower voltages. Thus, even if a given voltage level is sufficient for achieving the running torque, the machine may not start. This method of trying to control the speed is best suited for loads that require very little starting torque, but their torque requirement may increase with speed.

![Figure: Speed-torque curves: voltage variation](image)

Above figure also shows a load torque characteristic | one that is typical of a fan type of load. In a fan (blower) type of load, the variation of torque with speed is such that \( T \sim \omega^2 \). Here one can see that it may be possible to run the motor to lower speeds within the
range \( n_s \) to \((1 - s)n_s\). Further, since the load torque at zero speed is zero, the machine can start even at reduced voltages. This will not be possible with constant torque type of loads. One may note that if the applied voltage is reduced, the voltage across the magnetising branch also comes down. This in turn means that the magnetizing current and hence flux level are reduced. Reduction in the ux level in the machine impairs torque production (recall explantions on torque production), which is primarily the explanation for above fig.

If, however, the machine is running under lightly loaded conditions, then operating under rated flux levels is not required. Under such conditions, reduction in magnetizing current improves the power factor of operation. Some amount of energy saving may also be achieved.

Voltage control may be achieved by adding series resistors (a lossy, inefficient proposition), or a series inductor / autotransformer (a bulky solution) or a more modern solution using semiconductor devices. A typical solid state circuit used for this purpose is the AC voltage controller or AC chopper. Another use of voltage control is in the so-called `soft-start' of the machine. This is discussed in the section on starting methods.

### 2 Rotor resistance control

The expression for the torque of the induction machine. Clearly, it is dependent on the rotor resistance. Further, eqn. 18 shows that the maximum value is independent of the rotor resistance. The slip at maximum torque eqn. 17 is dependent on the rotor resistance. Therefore, we may expect that if the rotor resistance is changed, the maximum torque point shifts to higher slip values, while retaining a constant torque. Figure shows a family of torque-speed characteristic obtained by changing the rotor resistance.

Note that while the maximum torque and synchronous speed remain constant, the slip at which maximum torque occurs increases with increase in rotor resistance, and so does the starting torque. Whether the load is of constant torque type or fan-type, it is evident
that the speed control range is more with this method. Further, rotor resistance control could also be used as a means of generating high starting torque.

For all its advantages, the scheme has two serious drawbacks. Firstly, in order to vary the rotor resistance, it is necessary to connect external variable resistors (winding resistance itself cannot be changed). This, therefore necessitates a slip-ring machine, since only in that case rotor terminals are available outside. For cage rotor machines, there are no rotor terminals. Secondly, the method is not very efficient since the additional resistance and operation at high slips entails dissipation.

The resistors connected to the slip-ring brushes should have good power dissipation capability. Water based rheostats may be used for this. A `solid-state' alternative to a rheostat is a chopper controlled resistance where the duty ratio control of of the chopper presents a variable resistance load to the rotor of the induction machine.

![Figure: Speed-torque curves: rotor resistance variation](image)

3 Stator frequency control

The expression for the synchronous speed indicates that by changing the stator frequency also it can be changed. This can be achieved by using power electronic circuits called
inverters which convert dc to ac of desired frequency. Depending on the type of control scheme of the inverter, the ac generated may be variable-frequency-fixed-amplitude or variable-frequency-variable-amplitude type. Power electronic control achieves smooth variation of voltage and frequency of the ac output. This when fed to the machine is capable of running at a controlled speed. However, consider the equation for the induced emf in the induction machine.

\[ V = 4.44N\phi_m f \]

where N is the number of the turns per phase, \( \phi_m \) is the peak flux in the air gap and f is the frequency. Note that in order to reduce the speed, frequency has to be reduced. If the frequency is reduced while the voltage is kept constant, thereby requiring the amplitude of
induced emf to remain the same, \( u_x \) has to increase. This is not advisable since the machine constant which implies that voltage must be reduced along with frequency. The ratio is held constant in order to maintain the \( u_x \) level for maximum torque capability. Actually, it is the voltage across the magnetizing branch of the exact equivalent circuit that must be maintained constant, for it is that which determines the induced emf. Under conditions where the stator voltage drop is negligible compared the applied voltage, from above eqn. is valid.

In this mode of operation, the voltage across the magnetizing inductance in the 'exact' equivalent circuit reduces in amplitude with reduction in frequency and so does the inductive reactance. This implies that the current through the inductance and the \( u_x \) in the machine remains constant. The speed torque characteristics at any frequency may be estimated as before. There is one curve for every excitation frequency considered corresponding to every value of synchronous speed. The curves are shown below. It may be seen that the maximum torque remains constant.

![Figure: Torque-speed curves with \( E=f \) held constant](image)

Figure : Torque-speed curves with \( E=f \) held constant
This may be seen mathematically as follows. If $E$ is the voltage across the magnetizing branch and $f$ is the frequency of excitation, then $E = kf$, where $k$ is the constant of proportionality. If $\omega = 2\pi f$, the developed torque is given by

$$T_{E/f} = \frac{k^2 f^2}{\left(\frac{R'}{s}\right)^2 + (\omega L'_{tr})^2} \frac{R'}{s\omega}$$

If this equation is differentiated with respect to $s$ and equated to zero to find the slip at maximum torque $s$, we get $s = \pm R'/\omega L'_{tr}$. The maximum torque is obtained by substituting this value into eqn. 23.

Equation 24 shows that this maximum value is independent of the frequency. Further, $s$ is independent of frequency. This means that the maximum torque always occurs at a speed lower than synchronous speed by a fixed difference, independent of frequency. The overall effect is an apparent shift of the torque-speed characteristic as shown in fig.

Though this is the aim, $E$ is an internal voltage which is not accessible. It is only the terminal voltage $V$ which we have access to and can control. For a fixed $V$, $E$ changes with operating slip (rotor branch impedance changes) and further due to the stator impedance drop. Thus if we approximate $E=f$ as $V=f$, the resulting torque-speed characteristic shown in fig. is far from desirable.
Single Phase Induction Motor
9.1 INTRODUCTION

The characteristics of single phase induction motors are identical to 3-phase induction motors except that single phase induction motor has no inherent starting torque and some special arrangements have to be made for making it self starting. It follows that during starting period the single phase induction motor must be converted to a type which is not a single phase induction motor in the sense in which the term is ordinarily used and it becomes a true single phase induction motor when it is running and after the speed and torque have been raised to a point beyond which the additional device may be dispensed with. For these reasons, it is necessary to distinguish clearly between the starting period when the motor is not a single phase induction motor and the normal running condition when it is a single phase induction motor. The starting device adds to the cost of the motor
and also requires more space. For the same output a 1-phase motor is about 30% larger than a corresponding 3-phase motor.

The single phase induction motor in its simplest form is structurally the same as a poly-phase induction motor having a squirrel cage rotor, the only difference is that the single phase induction motor has single winding on the stator which produces mmf stationary in space but alternating in time, a poly phase stator winding carrying balanced currents produces mmf rotating in space around the air gap and constant in time with respect to an observer moving with the mmf. The stator winding of the single phase motor is disposed in slots around the inner periphery of a laminated ring similar to the 3-phase motor.

An induction motor with a cage rotor and single phase stator winding is shown schematically in Fig. 9.1. The actual stator winding as mentioned earlier is distributed in slots so as to produce an approximately sinusoidal space distribution of mmf.

PRINCIPLE OF OPERATION

Suppose the rotor is at rest and 1-phase supply is given to stator winding. The current flowing in the stator winding gives rise to an mmf whose axis is along the winding and it is a pulsating mmf, stationary in space and varying in magnitude, as a function of time, varying from positive maxi-mum to zero to negative maximum and this
pulsating mmf induces currents in the short-circuited rotor of the motor which gives rise to an mmf. The currents in the rotor are induced due to transformer action and the direction of the currents is such that the mmf so developed opposes the stator mmf. The axis of the rotor mmf is same as that of the stator mmf. Since the torque developed is proportional to sine of the angle between the two mmf and since the angle is zero, the net torque acting on the rotor is zero and hence the rotor remains stationary.

For analytical purposes a pulsating field can be resolved into two revolving fields of constant magnitude and rotating in opposite directions as shown in Fig. 9.2 and each field has a magnitude equal to half the maximum length of the original pulsating phasor.

![Fig. Representation of the pulsating field by space phasors.](image)

These component waves rotate in opposite direction at synchronous speed. The forward (anticlockwise) and backward-rotating (clockwise) mmf waves f and b are shown in Fig. In case of 3-phase induction motor there is only one forward rotating magnetic field and hence torque is developed and the motor is self-starting. However, in single phase induction motor each of these component mmf waves produces induction motor action but the corresponding torques are in opposite direction. With the rotor at rest the forward and backward field produce equal torques but opposite in direction and hence no net torque is developed on the motor and the motor remains stationary. If the forward and backward air gap fields remained equal when the rotor is revolving, each of the component fields would produce a torque-speed characteristic similar to that of a poly phase induction motor with negligible leakage impedance as shown by the dashed curves f and b in Fig.
The resultant torque-speed characteristic which is the algebraic sum of the two component curves shows that if the motor were started by auxiliary means it would produce torque in what-ever direction it was started.

Fig. . Torque-speed characteristic of a 1-phase induction motor based on constant forward and backward flux waves.

In reality the two fields, forward and backward do not remain constant in the air gap and also the effect of stator leakage impedance can’t be ignored. In the above qualitative analysis the effects of induced rotor currents have not been properly accounted for.

When single phase supply is connected to the stator and the rotor is given a push along the forward rotating field, the relative speed between the rotor and the forward rotating magnetic field goes on decreasing and hence the magnitude of induced currents also decreases and hence the mmf due to the induced current in the rotor decreases and its opposing effect to the forward rotating field decreases which means the forward rotating field becomes stronger as the rotor speeds up. However for the backward rotating field the relative speed between the rotor and the backward field increases as the rotor rotates and hence the rotor emf increases and hence the mmf due to this component of current increases and its opposing effect to the backward rotating field increases and the net backward rotating field weakens as the rotor rotates along the forward rotating field. However, the sum of the two fields remains constant since it must induce the stator counter emf which is approximately constant if the stator leakage impedance drop is negligible. Hence, with the rotor in motion the torque of the forward field is greater and that of the backward field is less than what is shown in Fig. The true situation being as is shown in Fig.
STARTING OF SINGLE PHASE INDUCTION MOTORS

The single phase induction motors are classified based on the method of starting method and in fact are known by the same name descriptive of the method. Appropriate selection of these motors depends upon the starting and running torque requirements of the load, the duty cycle and limitations on starting and running current drawn from the supply by these motors. The cost of single phase induction motor increases with the size of the motor and with the performance such as starting torque to current ratio (higher ratio is desirable), hence, the user will like to go in for a smaller size (hp) motor with minimum cost, of course, meeting all the operational requirements. However, if a very large no. of fractional horsepower motors are required, a specific design can always be worked out which might give minimum cost for a given performance requirements. Following are the starting methods.

(a) Split-phase induction motor. The stator of a split phase induction motor has two windings, the main winding and the auxiliary winding. These windings are displaced in space by 90 electrical degrees as shown in Fig. 9.5 (a). The auxiliary winding is made of thin wire (super enamel copper wire) so that it has a high R/X ratio as compared to the main winding which has thick super enamel copper wire. Since the two windings are connected across the supply the current $I_m$ and $I_a$ in the main winding and auxiliary winding lag behind the supply voltage $V$, $I_a$ leading the current $I_m$ Fig. 9.5(b). This means the current through auxiliary winding reaches maximum value first and the mmf or flux due to $I_a$ lies along the axis of the auxiliary winding and after some time ($t = \theta/w$) the current $I_m$ reaches maximum value and the mmf or flux due to $I_m$ lies along the main winding axis. Thus the motor becomes a 2-phase unbalanced motor. It is unbalanced since the two currents are not exactly 90 degrees apart. Because of these two fields a starting
The capacitor start induction motor is also a split phase motor. The capacitor of suitable value is connected in series with the auxiliary coil through a switch such that $I_a$, the current in the auxiliary coil leads the current $I_m$ in the main coil by 90 electrical degrees in time phase so that the starting torque is maximum for certain values of $I_a$ and $I_m$. This becomes a balanced 2-phase motor if the magnitude of $I_a$ and $I_m$ are equal and are displaced in time phase by 90° electrical degrees. Since the two windings are displaced in space by 90 electrical degrees as shown in Fig. 9.6 maximum torque is developed at start. However, the auxiliary winding and capacitor are disconnected after the motor has picked up 75 per cent of the synchronous speed. The motor will start without any humming noise. However, after the auxiliary winding is disconnected, there will be some humming noise.